

# From Clouds to Planet Systems Formation and Evolution of Stars and Planets

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## Abstract

*The discovery of more than hundred extrasolar planet candidates challenges our understanding of star and planet formation. Do we have to modify the theories that were mostly developed for the solar system in order to understand giant planets orbiting their host stars with periods of a few days? Or do we have to assume particular circumstances for the formation of the Sun to understand the special properties of the solar system planets? I will review the theories of star and planet formation and outline processes that may be responsible for the diversity of planetary systems in general. I will discuss two questions raised by extrasolar planets: (1) the formation of Pegasi-planets and (2) the relation between discovered extrasolar planets and the metallicity of their host stars. Finally I will discuss the role of migration in planet formation and describe three tests in order to distinguish whether planets migrated long distances or formed near their final orbits.*

## 1 Witnessing the Discovery

What happened to the theory of star and planet formation, when almost ten years ago, in October 1995, Mayor and Queloz (1995) announced that they had found a planet, in a 4 day orbit around the 5th magnitude star 51 Pegasi? Theory at this time was preparing for the discovery of extrasolar planets in orbits around common, main-sequence stars. Yet the first discoveries seemed to be well in the future, not to be expected before the start of the new, the third millennium.

### 1.1 Observations: 12 years, 21 stars - no planet (yet)

In August that year, planet searchers from the University of British Columbia, led by Gordon Walker, had published their 12 year long effort to monitor 21

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solar-like stars for planetary companions. It was widely considered the first search having the required sensitivity to detect planets. And as everybody considered self-evident, the easiest one to detect would be a Jupiter-like planet in a Jupiter-like orbit of approximately 12 years period. After all that was the case for the solar system put to 10 parsec, the typical distance of a nearby star.

When Walker et al. (1995) published that they had not found any planet<sup>1</sup> and gave an upper limit of less than 1 in 10 stars having a ‘Jupiter’, the formation theorists started to wonder about the result. It was generally considered that solar system formation theory was not yet complete. But should the solar system be more special after all, then the Sun was compared to other stars? I.e. a typical member of the galactic minority of single stars that is outnumbered by a factor of about 3 by stars that are in multiple systems.

The Wolszczan and Frail (1992) finding of planets in orbit around the Pulsar PSR-B1257+12, had been confirmed by Wolszczan (1994) using modifications of the planetary orbits by mutual perturbations of the inferred planets themselves. Hence apparently planet formation was a rather ubiquitous process also occurring in one of the very exotic environments that the millisecond pulsar had provided during its long history.

## 1.2 Theory: not quite in place

That October theorists had assembled on the Island of Hawaii to discuss extrasolar planets and their formation. The discovery seemed like a reality shock. I still remember the faces of other theorists after the message of the discovery of 51 Pegasi was put on an overhead projector at the end of the extrasolar planets session. We realised that we were on the wrong side of the globe. The discovery was announced in Florence, close to Galileo’s last home. Theory was taken by surprise. A ‘Jupiter’ at 1/100 of Jupiter’s orbit, a few stellar radii from its host star was a bewildering thing. It was not that anyone had proven or even tried to prove that giant planets could not form close to their stars. Apparently nobody had even thought about it. Everybody wondered how this could have happened and there was at the same time concentrated thinking: how could it form? I am sure, that as we travelled home we all had an idea how to make it. When I stepped out of the plane in Frankfurt, I had convinced myself that it actually would be easy to form 51 Peg’s planet and planned the calculations to prove that. Similar events must have happened for many people because within a year many ideas for the formation of 51 Pegasi b were published.

I am telling this because it is exciting and also because it helps to understand the diversity of ideas that developed after the discovery of 51 Pegasi b. Before that, there was a fairly detailed framework for the understanding of the formation of the solar system. It was a step-wise approach, tied into the data from the history of the solar system. Supposedly it could be generalised to other

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<sup>1</sup>Although  $\gamma$  Cep was a special case where the planetary interpretation could not be rejected with certainty, cf. Walker et al. (1992). In the meantime two of the stars of Gordon Walker’s list are known to host planets:  $\epsilon$  Eridani Hatzes et al. (2000) and  $\gamma$  Cephei A Hatzes et al. (2003). Twelve years of data were not enough.

systems. But it had and has its open questions. Hence research focused on closing the holes for the solar system where the data was rich. An understanding of a general theory of planet formation was only beginning. As the pioneering work for understanding planet formation around stars of various masses I recommend a series of three papers: Nakano (1987, 1988a,b). Nakano discusses the limitations for planet formation based on growth time-scales and nebula and stellar lifetime in the framework of the *Kyoto*-model.

Just before the discovery Jack Lissauer gave a talk on the *Diversity of plausible planetary Systems* Lissauer (1995). It is a careful assessment and weighting of the uncertainties trying to focus on what can be expected for other systems from what we have learned for the solar system. I think every person interested in what could be expected from theory will find the overview there.

## 2 Planet Formation Theory

### 2.1 The ‘Original’ Solar System Perspective

So there is the classical picture based on the formation of the solar system until 1995 and the diversity of ideas thereafter, with a manifold of scenarios for planet formation. Many of them probably inconsistent with solar system formation or of unknown relation to the solar system.

To get an overview over the classical picture of solar system formation, I recommend the review article of Hayashi’s Kyoto-group Hayashi et al. (1985), where the stage is set for modern theories<sup>2</sup> — planets forming in a circumstellar, protoplanetary disk of gas and dust, that originates from an interstellar cloud together with the host (proto)-star. The article discusses the key physical processes and outlines a complete picture of solar system formation. The overall picture is basically unchanged up to now, but some of the numbers have been corrected and gaps in the argumentation have been closed. To get a more quantitative view, especially on terrestrial planet formation and a concept to resolve the issue of the long growth time of the cores of giant planets I recommend Jack Lissauer’s Lissauer (1993) article on *Planet Formation* in *Annual Reviews of Astronomy and Astrophysics*. It is written after a half-year long get-together of essentially all researchers on planet formation in 1992, at Santa Barbara. It is a superb review and covers the situation of planet formation theory at that pre-51 Peg-time.

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<sup>2</sup>v. Weizsäcker Weizsäcker (1943) distinguishes three mayor groups of planet formation theories: A. planets formed from a uniformly rotating mass, that filled the space of the present planetary system (Kant, Laplace). B. Planets formed from a tidal-wave [*Flutwelle*] that was excited by a star passing the Sun (Chamberlin, Moulton). C. Planets formed from an irregularly shaped nebula-filament (Nölke). I consider work following the *nebula hypothesis* (A.) supplemented by the *planetesimal hypothesis* as modern formation theories

## 2.2 After 51 Pegasi — Forget the Solar system?

With the ‘discovery shock’ many researches apparently felt that the solar system approach was ‘dead’ and invented numerous schemes to rapidly explain the new object. A year after the discovery it was still a close race between the number of discovered planets and the number of new theories to explain them.

New processes were invoked — often rather ad hoc — to explain the unexpected properties of 51 Peg as well as the growing diversity in the observed exoplanet population. Some of the new processes apparently played no or only a minor role in the solar system. The unpredicted properties of extrasolar planets — giant planets in orbits with periods of a few days, eccentricities much larger than any of the Sun’s planets, and many giant planets with orbital radii much smaller than Jupiter’s — prompted strong comments:

Upon the discoveries, theorists have lost the understanding of the formation of the solar system,

as Pavel Artymovic put it at the IAU symposium 202 at the Manchester general assembly in 2000 or more drastically:

Forget the solar system!,

as a well known astrophysicist recommended off the records in 2002. Are we witnessing that the copernican principle is finally failing in cosmogony? Is the formation of our own home-system fundamentally different from the typical planet formation process in the galaxy? Are special processes or unlikely circumstances required to explain the properties of the Solar System?

## 3 Introduction - Trying to solve a big problem

The approach I will follow to look at the problem and possibly contribute to answer these questions is not to describe the diversity of theories and ideas but focus on a simplified picture that is closely linked to basic physical principles. Essentially to the equations that describe the fluid-dynamics of a mixture of gas and dust supplemented by the theory of planetesimal growth.

The idea is to provide understanding of a theoretical backbone that is unlikely to suffer refutation, unless one of its key assumptions turns out to be invalid.

Those assumptions are

1. There exists an angular momentum transfer process to separate mass and angular momentum during protostellar collapse,
2. Dust-growth in the protoplanetary nebula leads to km-size planetesimals on a time-scale that is fast compared to nebula-evolution,
3. The basic properties of stars and planets can be described with spherical symmetry.

I will not discuss many results in the literature based on my judgment of how close they are to a deductive, more theoretical physics approach. It is not because they may not be relevant but because they are usually snap-shot studies of a particular process without a clear justification of previous or later phases of star and/or planet formation. Often unknown physical processes are parameterised to make the respective models solve-able.

So I emphasise parts of the problem that can be solved with relatively high reliability but neglect the addition of processes that will occur in reality but can not be reliably quantified. Such, the picture given here will not address many aspects of the problem that are needed to understand all the properties of star planet systems but what is discussed should hold, although it may turn out to be of minor relevance for the big picture.

The analogy for the Sun would be to just discuss the global properties like mass, luminosity, temperature and age but ignore surface effects like spots and other magnetic activity and global effects like differential rotation and globally relevant circulation patterns inside the convection zone and granulation.

## 4 The Plan

We will start our considerations with interstellar clouds and proceed to planets in three steps:

1. star formation as collapse of a gravitationally unstable cloud fragment,
2. early evolution of the star and a plausible circumstellar, protoplanetary nebula,
3. planet formation in the protoplanetary nebula.

I will discuss protostellar collapse and early stellar evolution with detailed solutions for the spherical problem for masses down to the brown dwarf regime. This is analogous to describing stellar evolution to and from the main sequence, with the important difference that due to the collapse-origin of stars fluid dynamical processes are important. Accretion-flows determine the luminosity, structure and evolution during the earliest phases that last approximately a Ma (1 Ma = 1 Million years).

A similar first principles approach is presently not feasible for the formation of the protoplanetary nebula. Hence I will briefly describe the ongoing efforts and the fall back to a pragmatic approach to construct plausible protoplanetary nebulae for planet formation studies. The resulting models then give estimates for the nebulae conditions that then can be used as starting point for the construction of planet formation models.

In the third step, i.e. planet formation I will return to a more basic principle oriented approach. The theory of planetesimal accretion will be combined with spherical models of protoplanets that consist of a core and a gaseous envelope that are embedded in a plausible protoplanetary nebula. These models will provide masses, luminosities and accretion-histories for planets. Finally I

will discuss some applications, including observational test of planet formation theory.

For convenience and consistency I will mostly use my own models that are applicable to stars, brown dwarfs and planets and are based on the same set of basic physical equations and identical descriptions of the microphysics. They all use the same *constitutive relations*, i.e. equations of state and opacities. The system of equations is calibrated to the Sun and observationally tested by the solar convection zone and RR-Lyrae light-curves Wuchterl and Feuchtinger (1998); Wuchterl and Tscharnuter (2003).

## 5 From Clouds to Stars

### 5.1 Clouds

Stars form from molecular clouds. Most of the clouds' mass is in molecular hydrogen. Some of them can be seen as nearby dark clouds because the dust they contain obscures the light of milky-way stars behind them. In some of the dark clouds faint stars can be seen that have ages of a few Mio years. These *T-Tauri* stars have not yet ignited hydrogen burning and will evolve towards the main sequence in a few 10 Ma. Hence this evolutionary phase is often referred to as the pre-main sequence phase. Only when their nuclear reactions will produce energy with a sufficient rate to balance the energy that is radiated from their surfaces, the process of gravitational contraction stops. Once that balance is reached typical stars will stay on the main sequence with approximately constant luminosity and surface temperature for  $\sim 10^9$  years.

Star formation preferentially occurs in the spirals arms of the galaxy. Giant molecular clouds there have masses of 1 Mio solar masses and fragment into substructures that finally lead to the sub-collapses of sub-fragments, that typically result in a cluster of stars. Star formation is a multiple scale process reaching from sizes of 100 pc clouds with subunits referred to as clumps down to smallest structures termed cores of 0.1 pc and masses comparable to the stellar ones, i.e. of typically a solar mass. Galactic tides, magnetic fields and irregular motion (turbulence) play a role on larger scales but ultimately star formation is a competition between the support by thermal pressure and the inward pull of gravity. Once gravitational collapse starts and the cloud-cores shrink, rotation becomes more and more important. As the sizes shrink, conservation of angular momentum leads to a spin up even for initially slow rotating structures. The common outcome are clusters of stars, multiple systems and star-disk systems. That way the classical angular momentum problem — even a slowly rotating cloud would lead to a hypothetical stellar embryo rotating much faster than breakup speed — is most likely solved by redistribution of angular momentum between the components.

I focus on the physics of star formation that is most relevant for planet formation. That is most importantly the properties of the new-born protostar and the protoplanetary nebula as well as the time-scales for the final collapse

and early stellar evolution. A general review of the *physics of star formation* has recently given by Larson (2003). Two stages of cloud collapse may be discerned:

1. fragmentation of the cloud,
2. collapse and accretion of cloud fragments.

Instead of discussing the fragmentation process of molecular clouds I will assume that it leads to fragments of all masses down to the *opacity limit* of fragmentation, which is estimated to be at approximately  $0.01 - 0.007 M_{\text{Sun}}$  or  $7-10 M_{\text{Jupiter}}$ <sup>3</sup>. Below the opacity limit further fragmentation into still smaller mass units is likely made more difficult or impossible. That is due to the temperature increase which is a consequence of the dense fragments becoming opaque for light of all wavelengths that are capable to carry out energy and hence cool the collapsing clouds that are heated by gravitational self-compression. Indeed stars may have their masses determined directly by those of the prestellar cloud cores. That is suggested by the fact that the distribution of masses or ‘initial mass function’ (IMF) with which stars are formed appears to resemble the distribution of masses of the prestellar cores in molecular clouds (Larson (2003), for discussion).

To determine the physical properties of the final fragments, that just became gravitationally unstable, we consider the balance of gravity and thermal pressure in a cloud-core of given temperature,  $T$  and mass-density (or density for short)  $\varrho$ . Such a fragment is unstable to perturbations, i.e. density fluctuations, if its size is or volume contains a mass that is larger than the *Jeans-mass*:

$$M_{\text{Jeans}} = \rho \lambda_{\text{Jeans}}^3, \quad (1)$$

with the *Jeans-length*,

$$\lambda_{\text{Jeans}}^3 = c_T \sqrt{\frac{\pi}{G \varrho}} \quad (2)$$

where  $G$  is the gravitational constant,  $c_T = \sqrt{P/\varrho} = \sqrt{kT/m}$ , the isothermal sound speed for an ideal gas of particles with mean mass  $m$ , with the Boltzman constant  $k$ . The other constant factors depend somewhat on the details of the assumed geometry of the cloud and the perturbations respectively density fluctuations assumed in the cloud<sup>4</sup>.

We will restrict our discussion of star-formation to the discussion of star-formation from gravitationally unstable — to be more specific Jeans-unstable — cloud cores as an idealisation of the smallest cloud fragments. To be more precise we will mostly discuss star formation from cloud fragments that are actual

<sup>3</sup> $1 M_{\text{Sun}} = 1.989 \cdot 10^{30} \text{ kg}, 1 M_{\text{Jupiter}} = 1.899 \cdot 10^{25} \text{ kg}, 1 M_{\text{Earth}} = 5.974 \cdot 10^{24} \text{ kg}, 1 M_{\text{Jupiter}} = 0.95 \cdot 10^{-3} M_{\text{Sun}}, 1 M_{\text{Jupiter}} = 317.8 M_{\text{Earth}}$ . I will use  $0.001 M_{\text{Sun}}$  and  $M_{\text{Jupiter}}$  as approximately equal.

<sup>4</sup>Planar waves are used in the case given. A discussion of variants of the Jeans instability and more rigorous treatments that avoid the *Jeans-swindle* of assuming an unperturbed state that is not an equilibrium solution is given in Larson (2003) or text-books as Kippenhahn and Weigert (1990)

rigorous solutions for the equilibria of self-gravitating isothermal clouds with spherical symmetry that are embedded into a medium of finite, constant pressure (the ambient larger cloud superstructure). Such self-gravitating equilibrium gas-spheres are called Bonnor-Ebert-spheres, Bonnor (1956); Ebert (1957). Such theoretical constructs do actually exist in the sky: the structure of a relatively nearby dark-cloud, the Bok-globule Barnard 68, has been found to closely match a Bonnor-Ebert sphere, Alves et al. (2001).

The neglect of rotation, magnetic fields and turbulence as well as the use of spherical symmetry, that lead to the classical Jeans-picture with thermal pressure and gravity as the only players, may seem somewhat restrictive, but we follow Larson (2003) in noting that

... the Jeans length and mass are still approximately valid even for configurations that are partly supported by rotation or magnetic fields, as long as instability is not completely suppressed by these effects. Thus, if gravity is strong enough to cause collapse to occur, the minimum scale on which it can occur is always approximately the Jeans scale, and structure is predicted to grow most rapidly on scales about twice the Jeans scale.

## 5.2 Clouds and Stars

We have looked at the clouds in the beginning. Let us now look at the outcome of the formation process, the products of the cloud collapse: stars, brown dwarfs and planets, that we will refer to as *celestial bodies* for short. Basic properties that emphasise similarities are summarised in Tab. 1. What are the common

Stars	Brown Dwarfs	Planets
gas spheres	gas spheres	often gas spheres
self-gravitating	self-gravitating	self-gravitating
self-luminous	self-luminous	often self-luminous
often nuclear fusion	often nuclear fusion	heavy element enriched
main-sequence: luminosity balanced by nuclear burning	initial nuclear burning; luminosity never balanced	no nuclear burning according to historical practice and IAU working definition

Table 1: Basic properties of stars, brown dwarfs and planets.

physical principles of these celestial bodies? Arguable the most important one that distinguishes them from other common objects in the universe like rocks, plants, animals and cars is the importance of gravity. Newtonian gravity is described by the Poisson-equation, that in spherical symmetry can be integrated once and written in the form:

$$\frac{dM_r}{dr} = 4\pi r^2 \rho, \quad (3)$$

where  $\varrho$  is the density and the mass interior to radius  $r$  for a radial density distribution  $\varrho(r)$  is obtained by the integration mentioned above as:

$$M_r = \int_0^r 4\pi r'^2 \varrho dr'. \quad (4)$$

The corresponding gravitational force that acts on the structure at distance  $r$  is:

$$F_{\text{grav}} = -\frac{GM_r}{r^2} \varrho. \quad (5)$$

This leads us to one of the most important principles of astrophysics, namely the force-balance between gravity and pressure(-gradients) that governs the structure of gravitating gas-spheres throughout most of their life, namely hydrostatic equilibrium:

$$\frac{dP}{dr} = F_{\text{grav}} = -\frac{GM_r}{r^2} \varrho. \quad (6)$$

Pressure has entered the considerations and hence we have to specify an equation of state that relates pressure and density. For fully degenerate objects, as white dwarfs and neutron stars, and to some extent for planets, when they behave as liquids, the dominating pressure dependence is on density. Then our equations for the structure would be completed by specifying the pressure as a function of density alone. In general, however, and in any case for high temperatures, the pressure depends on density *and* temperature, as, e.g. in an ideal gas with particles of mean mass  $m$ :

$$P = \frac{\varrho}{m} kT, \text{ often written as } P = \frac{R_{\text{gas}}}{\mu} \varrho T. \quad (7)$$

The first version uses the mean particle mass directly and hence the Boltzman constant appears explicitly, the second version uses quantities for a mole of an Avogadro-number,  $N_A$  of particles, with the molar gas constant  $R_{\text{gas}} = kN_A$  and their mean molar mass  $\mu$ . Two important new quantities have entered our considerations via the pressure equation of state (Equ. 7):

1. the elemental and chemical composition, i.e. its elemental abundances and the respective chemical state: whether they are in molecules, neutral or ionised and the number of free electrons in the latter case. The mean particle mass  $m(\varrho, T)$  is changing accordingly.
2. The temperature, to be more explicit, the radial temperature distribution  $T(r)$  of a celestial body.

The chemical composition (number of particles as  $\text{H}_2$ ,  $\text{H}$ ,  $\text{e}^-$ , ...) and hence  $m(\varrho, T)$  can be determined by thermodynamic calculations for given elemental composition, density and temperature. Such calculations also give the pressure equation of state as well as other equations of state, as specific heats or the adiabatic gradients. What remains — apart from solving the whole thing — is to determine the temperature.

Here the self-luminous nature comes into the play. Because the celestial bodies usually radiate heat into space their temperature — the surface value and the interior temperature distribution  $T(r)$  — has to be calculated from an energy budget. That is the difference between the amount of energy that is radiated into space per unit time — their luminosity — and whatever amount of thermal energy is generated or available as a heat-reservoir in their interior.

The ratio between the amount of energy in the interior, and the luminosity of the celestial body can be used to calculate a time-scale for the change of the energy content. That results in the thermal, cooling, or Kelvin-Helmholtz time-scale:

$$\tau_{\text{KH}} = \frac{E_{\text{therm}}}{L} = \frac{M\bar{c}_V\bar{T}}{L} \sim \frac{1}{2} \frac{GM^2}{RL}. \quad (8)$$

Where we have ignored thermodynamical details and estimated the thermal energy of the celestial body by, first, using a mean specific heat  $\bar{c}_V$  and a mean temperature  $\bar{T}$ , and, then, by half the gravitational energy of a sphere of mass  $M$  and radius  $R$ . A valid approach for gas-spheres in hydrostatic equilibrium.

Due to the self-luminosity celestial bodies change their thermal energy content — in the absence of sufficient energy sources — and hence must evolve on the Kelvin-Helmholtz time-scale. That time-scale is of the order of a Ma for the objects under consideration.

The determination of the luminosity and the temperature leads to the introduction of an energy equation that contains the important energy transfer processes. For celestial bodies these are:

1. *radiative transfer*, that not only transfers heat through the interior but, in the end brings information from the photospheres to the observers telescopes,
2. *heat conduction* that usually is treated formally together with radiative transfer, and
3. *convection* i.e. energy transfer by small and medium scale gas motion, turbulent or otherwise that does not lead to large scale motion, restructuring the object or disobeying the overall hydrostatic equilibrium.

The short story of energy transfer is that the luminosity of an object is determined by its surface area and the amount of radiation that can be emitted by a unit surface area per unit time, well approximated by and equal to  $\sigma T^4$  in the black body case. For the *effective temperature*,  $T_{\text{eff}}$  that holds exactly, i.e.  $L = 4\pi R_\tau^2 \sigma T_{\text{eff}}^4$  for appropriately defined surface radius  $R_\tau$  from which the photons typically can travel directly to the observer. The interior temperature structure,  $T(r)$  is then determined by the surface temperature, and the temperature-increase or gradient, that is controlled by the efficiency of the dominating transfer process. Generally efficient energy transfer leads to a small temperature gradient for given luminosity and hence a moderate temperature increase towards the centre. Even almost constant temperature is a good approximation in case of very efficient radiative transfer or heat conduction or

almost zero luminosity. In the opposite extreme if all other energy transfers processes are inefficient, convection takes over, and essentially limits the temperature gradients to the adiabatic values. Or more precisely to an isentropic structure with the temperature gradients being such that the specific entropy is constant throughout a convective region. In that case the temperature gradient with respect to pressure<sup>5</sup> takes a particularly simple form:

$$\frac{d \ln T}{d \ln P} =: \nabla = \nabla_s := \left( \frac{\partial \ln T}{\partial \ln P} \right)_s. \quad (9)$$

Following the notation of classical stellar structure theory,  $\nabla$  denotes the gradient along the structure  $T(r), P(r)$  of a celestial body, whereas  $\nabla_s$  is the respective slope along an adiabat (or more precisely isentrope), i.e. a thermodynamical property of the particular material under consideration. A simpler way to put this (Equ. 9) is

$$\frac{ds}{dr} = 0, \quad (10)$$

where  $s$  is the specific entropy. The simplicity is paid for by the introduction of another equation of state, e.g.  $s(T, P)$ .

With the equations outlined above and setting more technical boundary conditions aside, we can calculate the structure of a celestial body — be it a planet, a brown dwarf or planet — for a given time when we know its structure at a previous time. The evolution is driven by the luminosity that changes the thermal content on the Kelvin-Helmholtz time-scale.

This simplified discussion must suffice for the present purpose. We note that the ideal gas is only a very rough approximation that already needs significant corrections in the solar interior. The interactions between particles in the gas that are responsible for that corrections become more and more important as typical densities increase and masses and temperatures decrease towards the brown dwarf and planetary domain. Jupiter and the Earth are much better approximated by a liquid than by a gas, but a similar argumentation involving more elaborate equations of state holds.

We are left with the fact that the determination of the temperature for self-luminous objects, that stars, brown-dwarfs and planets are, has led as to an evolutionary picture. The evolution is driven by the fact that these objects change their heat content by radiating into space. Hence following their evolution means following how they transfer heat from their interior into the surrounding, cold universe.

That very fact led Lord Kelvin to estimate the age of the Earth to be between 20 and 400 Ma from the temperature increase observed in deep mines<sup>6</sup> and made

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<sup>5</sup>I suggest to read pressure in a hydrostatic object as a coordinate. It is then monotonically decreasing with radius and hence runs from outside in.

<sup>6</sup>Because the temperature on Earth is increasing as we dig deeper there is apparently a temperature gradient. Because there is a gradient there must be a heat flux. That flux must

Eddington look for an energy source to sustain the luminosities of the Sun and the stars — subatomic energy as he called it.

For us it will prompt an important question: what was the initial thermal energy that the formation processes put into stars, brown dwarfs and planets? That input would determine their evolution. For stars until they reach the balance of nuclear energy generation and luminosity at the main sequence, for brown dwarfs and planets for the time-span until they have forgotten the details of their formation.

Given sufficient energy sources, the losses due to luminosity are balanced and the structure of a celestial body does not change unless its elemental composition changes due to nuclear reactions. That is the case for a star on the main sequence.

### 5.3 How to begin — Stellar Evolution as Initial Value Problem

Star formation involves phases that follow the gravitational instability of the cloud cores. That is an instability of the hydrostatic equilibrium. It can be shown that it continues to grow rapidly in the non-linear regime departing further and further from the initial (close-to) equilibrium conditions. The initial perturbations are rapidly forgotten. This leads to a vanishing role of the gas pressure (e.g. Kippenhahn and Weigert (1990)). Hence the subsequent evolution is dominated by gravity alone — a collapse with the cloud falling freely towards its centre. For the prevailing, isothermal cloud conditions that holds for about then orders of magnitude growth in density. In the absence of gas pressure the cloud would collapse to a point in a *free-fall time*<sup>7</sup>

$$\tau_{\text{ff}} = \sqrt{\frac{3\pi}{32G\varrho_0}}, \quad (11)$$

where  $\varrho_0$  is the constant (or mean) initial density of the cloud, when it becomes unstable.

Because the free-fall time is much smaller than the cooling time,  $\tau_{\text{KH}}$ , (Equ. 8) of a typical new born celestial body the collapse leaves a thermal imprint. The energy transfer processes are too slow to erase the  $T(r)$  structure that the collapse builds up. Hence it is unlikely that new-born celestial bodies, in particular stars have adjusted their thermal structure to the one required on the main sequence to balance the luminosity needs by appropriate nuclear energy production. Therefore we have an evolution from an initial thermal structure to the one on the main sequence — *that is the pre-main sequence evolution*.

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transfer heat from a reservoir. Given the size of the reservoir — the Earth — and the flux inferred from the gradient we can estimate the time that is needed to reduce the reservoir. That is the Kelvin-Helmholtz time-scale,  $\tau_{\text{KH}}$

<sup>7</sup>This corresponds to half an orbit with a semi-major axis of twice the radius of a constant density cloud with mass  $M$ . Imagine a very elongated ellipse with apastron at the cloud radius and the periastron approaching the cloud centre. Kepler's third law for a semi-major axes of half the cloud radius and the cloud mass as the primary mass, then gives the free-fall time as half the orbital period.

Early stellar evolution onto the main sequence is a gravothermal relaxation process from the thermal structure produced by the star formation process to the thermal structure determined by the balance of energy radiated into space from the stellar surface and energy generated by nuclear reactions in the stellar interior. If star formation produced young stars with thermal structures closely resembling those of main sequence stars there would be no or only a negligible pre-main sequence phase. Embedded objects would then start shining through their cocoons with main sequence stellar properties. If star formation resulted in a thermal structure of stars that is considerably different from the main sequence, a significant phase of gravothermal relaxation would be expected. Observations of young stars high above the main sequence prove the latter is the case. Those *classical T-Tauri stars* still show accretion-signatures (e.g.  $H_\alpha$ -emission, ‘veiling’ of the spectral lines and IR-excess) and cloud remnants in their vicinity (IR- and mm-emission of disks as well as residual circumstellar envelopes at least in many cases).

Since the evolution towards the stellar main sequence depends on the thermal structure provided by the stellar formation process, star formation has to be considered to determine the pre-main sequence evolution. An uncertainty in the stellar structure derived from a study of protostellar collapse causes an uncertainty in pre-main sequence evolution. A quantitative theory of star formation is therefore needed to provide the correct ‘initial’ thermal structure of stars to derive the pre-main sequence stellar structure and evolution as well as the stellar properties during that time.

## 5.4 Early stellar evolution theory

Present stellar evolution theory mostly deals with stars that are in hydrostatic equilibrium, where gas pressure balances gravity. The motion and inertia of stellar gas are usually neglected. The starting point of stellar evolution calculations is the early pre-main sequence phase. The mechanical structure there is determined by solving for the hydrostatic equilibrium. However, to obtain the thermal structure requires knowledge of still earlier evolutionary stages. Trying to obtain thermal information from the evolution in those embedded stages of star formation complicates the question even more because then the hydrostatic equilibrium cannot be used to determine the mechanical structure.

One important aim of star formation theory, which is still an unsolved problem, is therefore to provide the initial conditions for stellar evolution, i.e., the masses, radii and the internal structure of young stars as soon as they can be considered to be in hydrostatic equilibrium for the first time.

The calculation of appropriate starting conditions for stellar evolution is complicated by the fact that, in general, young stars still accrete mass. Therefore, the hydrostatic parts of young stars and their photospheres are more or less directly connected to circumstellar material being in motion due to mass-inflow and/or outflow. Moving circumstellar matter as well as the accretion process are by nature non-hydrostatic. Flows that contain both the hydrostatic protostellar core and the hydrodynamic accretion flow must be calculated by using at

least the equations of radiation hydrodynamics including convection and cause a wealth of technical difficulties.

Modelers of early stellar evolution and the pre-main sequence phase have therefore relied on simplified concepts to make the problem tractable. Originally, Hayashi et al. (1962) argued that the cloud collapse should be so fast that the fragment would evolve adiabatically to stellar conditions and the hydrostatic young star would appear as an isentropic sphere radiating at high luminosities, thus causing a fully convective structure. For a given stellar mass they would appear in the Hertzsprung-Russel-diagram on the almost vertical line defined for fully convective models of all luminosities — the Hayashi-track. Because Hayashi’s estimate for the collapse led to large radii, they should appear near the top of that line.

Larson (1969) showed that radiative losses during collapse are substantial and early collapse would proceed isothermally. Detailed models showed the necessity of a careful budgeting of the energy losses in the framework of radiation hydrodynamics (RHD) and demonstrated the high accuracy requirements for a direct calculation of the collapse, (Appenzeller and Tscharnuter (1974, 1975), Bertout (1976), Tscharnuter and Winkler (1979), Winkler and Newman (1980a,b), Tscharnuter (1987), Morfill et al. (1985), Tscharnuter and Boss (1993), Balluch (1991a,b,a) Kuerschner (1994) see Wuchterl and Tscharnuter (2003)). Modelers then looked for simplified, sometimes semi-analytical concepts to characterize the collapse and accretion flow with key parameters chosen in accord with values from detailed RHD-models of protostellar collapse or from properties of specific collapse solutions like the constant mass accretion rate for the self-similar singular isothermal sphere or those resulting from accretion disk models.

## 5.5 Strategies to determine ‘initial’ stellar structure

The studies of star formation have not resulted in an easy way to calculate star formation before beginning a pre-main sequence stellar evolution calculation. To obtain the initial stellar structure for such calculations the thermal structure produced by star formation has been approximated using different concepts to separate the early stellar evolution and the dynamics of protostellar collapse. The simplification strategies differ by how the complete problem is split into a quasi-hydrostatic ‘stellar’ and a hydrodynamic ‘accretion’ part, both in space and in time:

### 5.5.1 Quasi-hydrostatic, constant mass stellar evolution

Quasi-hydrostatic calculations use high luminosity initial conditions for a given stellar mass, i.e., with accretion assumed to have terminated or only causing negligible effects on the pre-main sequence. Once the mass is chosen, the initial entropy and radial entropy structure has to be specified before the initial model can be constructed. The choice of entropy essentially results in a value for the stellar radius. To be sure that the details of the star formation process, or more

precisely, the specific initial conditions do not influence such studies, the initial luminosities are chosen to be very high. If they are sufficiently high, the later evolution rapidly becomes independent of the earlier evolution.

Usually the internal thermal structure of the star (temperature or entropy profile) is assumed at a moment when dynamical infall motion from the cloud onto the young star is argued to have faded and contraction of the star is sufficiently slow (very subsonic) so that the balance of gravity and pressure forces, i.e., hydrostatic equilibrium (Equ. 6), accurately approximates the mechanical structure (pressure profile) of the star. The absolute ages associated with these states are obtained by the homological back-extrapolation of the so obtained initial hydrostatic structure to infinite radius. This leads to typical initial ages of  $\sim 10^5$  a, i.e., of the order of a free-fall time for a solar mass isothermal equilibrium cloud. At  $t > 10^6$  a the initially setup thermal structures are assumed to have decayed away sufficiently — as is the case in a familiar, non-gravitating thermal relaxation process after a few relaxation times — and the calculated stellar properties would then approximate well the properties of young stars which do form by dynamical cloud collapse in reality. This relaxation issue is somewhat complicated by the thermodynamic behaviour of self-gravitating non-equilibrium systems that stars are alike. However, it can be shown that the memory of the initial thermal structure is lost quickly, in some cases (Bodenheimer (1966), von Sengbusch (1968), Baraffe et al. (2002)), in particular if the star is initially *fully convective*.

A fully convective structure is considered to be the most likely result of the protostellar collapse of a solar mass cloud fragment, Hayashi (1961, 1966); Hayashi et al. (1962); Stahler (1988a) and hence is used as stellar-evolution initial condition.

Following this argumentation young star properties are now usually calculated from simple initial thermal structures without considering the gravitational cloud collapse: Chabrier and Baraffe (1997), D’Antona and Mazzitelli (1994), Forestini (1994), use  $n = 3/2$  polytropes to start their evolutionary calculations; Siess et al. (1997, 1999) also use polytropes; Palla and Stahler (1991)<sup>8</sup> found  $n = 3/2$  polytropes insufficient and use fully convective initial models.

### 5.5.2 Hydrostatic stellar embryo and parameterised accretion

To arrive at a more realistic description of the transition of cloud collapse to early stellar evolution the later parts of the accretion process have been modelled. The strategy is to start with a stellar embryo, i.e. with a hydrostatic structure of less than the final stellar mass. The remaining mass-growth is described by a separate accretion model. The argumentation for the central hydrostatic part is the same as above, but applied to the initial stellar embryo (protostellar core). Stahler (1988a), e.g., chose an embryo mass  $M_0 = 0.1 M_\odot$  after trial integrations for embryo masses  $> 0.01 M_\odot$ . Typically an embryo of  $0.05 M_\odot$  is chosen to be embedded in a steady accretion flow with given mass accretion rate. The

<sup>8</sup>initial conditions have been kept by the authors since then, e.g., Palla and Stahler (1992, 1993)

procedure is thought to reduce the ambiguity in the initial entropy structure by lowering the initial hydrostatic mass. The entropy added to the initial core is calculated self-consistently with the prescribed mass addition from the steady inflow due to disk/or spherical accretion. The lowered arbitrariness in the initial entropy structure (mass and initial entropy only have to be chosen for a small fraction of the mass and result in an initial radius for the initial hydrostatic core) is accompanied by the requirement of additionally specifying the mass accretion rate  $\dot{M}$  and the state of the gas at the cloud boundary. The key advantage, however, is that, due to the assumption of the steadiness of the accretion flow, the mathematical complications are reduced considerably by changing a system of partial differential equations into one of ordinary differential equations.

Stahler (1988a) followed this approach to discuss pre-main sequence stellar structure based on a study of steady protostellar accretion in spherical symmetry (Stahler et al. (1980a,b), SST in the following) and argued that the fully convective assumption should be valid for young stars below  $2 M_{\text{Sun}}$ . Stahler (1988b) discusses the history of initial stellar structure and the role of convection in young stars and summarizes (p. 1483): ‘This nuclear burning, fed by continual accretion onto the core of fresh deuterium, both turns the core convectively unstable and injects enough energy to keep its radius roughly proportional to its mass Stahler (1988b)’. But Winkler and Newman (1980a,b), who did a fully time dependent study of protostellar collapse, but excluded convection a priori, found a persistence of the thermal profile produced by the collapse and very different young star properties after the accretion had ended.

The fully convective assumption was discussed in Stahler (1988a) for the last time (p. 818) and although it was remarked that it had been only shown by SST for a solar mass it had been widely used for low mass stars of all masses based on a semi-analytical argumentation Stahler (1988a).

Unlike the competing Winkler and Newman study that neglected convection, SST however did not calculate the evolution before the main accretion phase and the transition towards the first hydrostatic core because ‘The detailed behaviour of these processes depends strongly on the assumed initial conditions, and there is little hope of observing this behaviour in a real system’.

Therefore instead of calculating this transient phase, SST assumed an initial hydrostatic core of  $0.01 M_{\text{Sun}}$  accreting matter in a quasi-steady way (p. 640). The entropy structure was assumed to be linear in mass and convectively stable inside the surface entropy spike due to the shock. Surface temperature was estimated to exceed values for Hayashi’s, Hayashi et al. (1962) forbidden zone and SST therefore assumed  $T_g = 3000$  (Stahler et al. (1980b), p. 234). Two values for dimensionless entropy gradient were tested (Stahler et al. (1980b), p. 235).

The entropy structure was then assumed to be linear in mass. After a comparison of the effects on later evolution for the different values of the assumed initial entropy gradient it was concluded that the differences had only small effect for the later evolution. The constant gradient assumption, however was never investigated.

### 5.5.3 Non-spherical accretion

Effects of non-spherical accretion on early stellar evolution have been taken into account in an analogous way as described in the previous section. The descriptions of accretion account for disks and magnetic fields in parameterized ways but only after an initial stellar or stellar embryo structure has been obtained as discussed above, see Hartmann et al. (1997), Siess and Forestini (1996), and Siess et al. (1997) for a discussion.

### 5.5.4 Summary on initial stellar structure

The theoretical knowledge about protostellar collapse and the pre-main sequence thus remained separated. On the one hand, modeling based on classical stellar structure theory was able to produce pre-main sequence tracks that could be related to observations if the question about the initial entropy distribution was put aside. On the other hand, models of the protostellar collapse revealed the entropy structure to be a signature of the accretion history, but were unable to provide pre-main sequence observables needed to be confronted with observations. That led to an *invited debate* at the IAU symposium 200 — *The Formation of Binary Stars*, that I tried to summarise in dialogue form (Wuchterl (2001a)).

Apparently there is a discrepancy about initial stellar structure between time-dependent radiative studies and time-independent convective studies. Recent advances, both in computational techniques and in the modeling of time-dependent convection, now make it possible to calculate the pre-main sequence evolution directly from cloud initial conditions by monitoring the protostellar collapse until mass accretion fades and the stellar photosphere becomes visible (Wuchterl and Tscharnuter (2003)).

## 6 Calculating protostellar collapse

Why is it that protostellar collapse calculations are not routinely used to determine the starting conditions for stellar evolution calculations? The reason lies in the very different physical regimes that govern the original clouds and the resulting stars as well as the fact that the transition between them is a dynamical one. Let us first compare the physical regimes. Cloud fragments are (1) quasi-homogeneous, i.e. the rim-centre density-contrast is less than 100, say, (2) cool, with temperatures of typically 10 K, (3) opaque for visible light but transparent at the wavelengths that are relevant for energy transfer — mostly controlled by dust radiating at the above temperature — and consequently, (4) isothermal.

On the other hand, stars, even the youngest ones are, (1) opaque, (2) compact, (3) hot, (4) non-isothermal, (5) with temperature gradients becoming so steep that convection is driven and plays a key role in energy transfer, (5) require non-ideal and often degenerate and generally elaborate (6) equations of state to describe ionisation and other processes, and consequently also require

(7) detailed calculations of atomic and molecular structure to quantify the corresponding opacities.

Simplifying approximations can be found that are valid in the cloud or the stellar regime, respectively. But to calculate the transition from clouds to stars a comprehensive system of equations has to be formulated. It has to contain both extreme regimes — clouds and stars. In addition, because the transition involves a gravitational instability of the cloud equilibrium, an equation of motion has to be used instead of the hydrostatic equilibrium (Equ. 6). It is the instability of that force-equilibrium that initiates the collapse in the first place. Following the instability the clouds collapse and their motion approaches a free-fall, i.e. gravity and pressure are nowhere near balance.

The equation of motion for a spherical volume  $V$  containing matter of density  $\varrho$  that is moving with velocity  $u$  can be written in the form:

$$\frac{d}{dt} \left[ \int_{V(t)} \varrho u \, d\tau \right] + \int_{\partial V} \varrho u (u_{\text{rel}} \cdot dA) = - \int_{V(t)} \left( \frac{\partial p}{\partial r} + \varrho \frac{GM_r}{r^2} \right) d\tau + C_M, \quad (12)$$

where the original force-balance of the hydrostatic equilibrium, (Equ. 6) now reappears as the first term on the right hand side. It has become a generally non-zero ‘source’ of momentum density  $\varrho u$ . The left side of equation Eq.12 is the total change of momentum for the volume  $V$  under consideration, i.e. the change inside the volume and what is transferred in and out across its surface  $\partial V$ . That changes are due to changes in the mass(-density) inside the volume and the acceleration due to the forces on the right hand side. Thus the above equation is the continuum version of Newtons second law written in reverse:

$$ma = F, \quad (13)$$

where the momentum can change due to both a change in mass and an acceleration due to the forces. For the forces we have the pressure gradient, gravity as in the hydrostatic equilibrium and  $C_M$ , the term describing the momentum coupling between matter and radiation. We can loosely speak of ‘radiation pressure’. To explain that term we briefly look at the case of an accreting object that produces a radiation field dominated by the energy generation due to the accretion process itself. Then, if the gas-pressure gradient is negligibly and gravity would balance  $C_M$  an object would accrete at the *Eddington-limit* with any larger radiation pressure becoming stronger than gravity and reverse the acceleration to outward and hence render more rapid accretion impossible.

If we put  $C_M$  to zero and require the explicit and implicit time-derivatives on the left side to be zero we recover (leaving the integral aside) Equ. 6, i.e. the hydrostatic equilibrium.

## 6.1 Inertia rules the collapse

The introduction of the equation of motion has important consequences. It introduces new time-scales. Globally things change on the free-fall time, Eq.(11)

for the collapse. Locally the typical time-scales for a significant change in a given volume are now comparable to the time a sound wave needs to cross that volume. This is the *dynamical time-scale*,

$$\tau_{\text{dyn}} = \frac{R}{c_s} = \frac{R}{\sqrt{\Gamma_1 \frac{P}{\rho}}} \approx \frac{R}{\sqrt{\frac{kT}{m}}}, \quad (14)$$

where  $R$  is the linear size of a typical region under consideration, e.g. the cloud radius or ultimately the stellar radius, and  $c_s$  is the isentropic (adiabatic) sound speed. Initially the dynamical time-scale is equal to the free-fall time for the initial equilibrium cloud. But as the stellar embryo takes shape and heats up the dynamical time-scales drop dramatically. Finally, for the mature main-sequence stars we are typically at the solar sound crossing time of 1.5 hours and minutes for oscillations in the upper layers, like the 5 minute oscillations used in helioseismology. It has to be kept in mind that the dynamical time-scale is an estimate based on the sound-speed. Hence events can be and are even faster in the hypersonic flows that appear in stellar collapse.

The physical ingredient preventing the rates of change to be even faster is the inertia of the matter. Unlike the *thermal inertia* that is controlled by the transfer processes and hence by the opaqueness of matter, inertia is universal. Therefore, if a cloud starts to collapse, the initial cloud structure imprints a time-scale onto the accretion process. The question then is how much energy the transfer processes can get out of the cloud during the collapse time.

## 6.2 Isothermal, 3D collapse

The equation of motion significantly complicates the mathematics and numerics compared to stellar evolution calculations that assume the hydrostatic equilibrium to hold. Yet the collapse is calculated almost routinely if additional assumptions are made about the cloud energetics. The simplest assumption is to use the initial cloud temperature. Such ‘isothermal’ calculations can be carried out without restrictive symmetry assumptions, i.e. in 3D and are accurate for the early stages of cloud collapse. They provide insight into the fragmentation process. However they also show that important processes happen at the transition to the non-isothermal phases. At this transition the cloud-centres get opaque and the temperature starts to rise, signalling the first, transient stop of the collapse process. That happens at some sufficiently high density, typically after an increase of about ten orders of magnitude from the original cloud conditions. One of the most important of these non-isothermal effects is likely to be the opacity-limit for the fragmentation process itself, the we briefly touched earlier. Hence while the isothermal calculation (and calculations with other special ‘equation of state assumptions’) can show the dynamics without symmetry restrictions they cannot answer the question of how much energy leaves the cloud during the collapse process and how much heat remains inside the star after accretion is completed.

### 6.3 The heat of young stars

To determine the temperature in the clouds during the non-isothermal phases of collapse, energy gains and losses have to be budgeted much in the way we have seen for the pre-main sequence phase above. The key differences result from the fact that now dynamics is under control of the time-scales — inertia rules —, not the energy transfer processes themselves. Therefore it becomes important how and how fast a transfer process can react on a change of the situation that is imposed by the dynamics. The collapsing clouds are mostly non-hydrostatic and sooner than later changing faster than the energy transfer processes can respond<sup>9</sup>. That requires a time-dependent treatment of all these processes. This is usually not necessary for later pre-main- sequence phases because the transfer processes control the changes leading to a quasi-equilibrium situation as described in Sect. 5.2.

In addition *radiative transfer* is complicated by the fact that the problem cannot be separated into an hydrostatic, very opaque part — the stellar interior — and a geometrically and optically thin part that emits the photons into the ambient space — the stellar atmosphere. A complete solution of the radiation transfer equation is needed.

Finally, *convection* is driven in the rapidly changing accretion flows and the rapidly contracting rapidly heated young stellar embryos. Convection is rapidly switched on or of requiring a description of how fast convective eddies are generated or vanish. That is unlike during the stellar (pre-) main-sequence evolution where changes are driven by chemical evolution or slow contraction and hence the convection pattern always has enough time to adjust itself to new interior structures and hence the time-independent convective energy flux of mixing length theory can be used. It is also unlike to stellar pulsations where convection zone are pre-existing and are modulated by the oscillation of the outer stellar layers. Convection in young objects is initiated in fully radiative structures and hence the rapid creation of a convection zone has to be covered by the theory — surprisingly a non-trivial requirement (cf. Wuchterl (1995b); Wuchterl and Feuchtinger (1998)). In short a time-dependent theory of convection is needed.

### 6.4 Beyond the equilibria of stellar evolution theory

Overall this results in a departure from the three mayor equilibria of stellar structure: hydrostatic equilibrium, radiative equilibrium und convective equilibrium. Instead of the equilibria budget equations have to be used. *Three* hydrodynamic equations to describe the motion of matter, *two* moment equations derived directly from the full radiative transfer equation that constitute the equations of motion for the radiation field, and *one* equation describing the generation and fading of convective eddies to calculate a typical kinetic energy for the eddies from which the convective flux can be obtained. The system includes mutual coupling terms that describe e.g. how moving matter absorbs and emits radia-

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<sup>9</sup>So fast that they play no rôle in parts of the flow for significant time-spans, leading to adiabatic phases.

tion or dissipating convective eddies create heat. Supplemented with the Poisson equation we arrive at the full set of the equations of fluid-dynamics with radiation and convection, also referred to as convective radiation-hydrodynamics. Altogether 7 partial differential equations instead of 4 ordinary ones for stellar evolution. To show the budget character of these equations more clearly and also because of advantages for solving them, we give the integral version of the equations that is valid for spherical volumes  $V$ :

$$\Delta M_r = \int_{V(t)} \varrho d\tau, \quad (15)$$

$$\frac{d}{dt} \left[ \int_{V(t)} \varrho d\tau \right] + \int_{\partial V} \varrho (u_{\text{rel}} \cdot dA) = 0, \quad (16)$$

$$\frac{d}{dt} \left[ \int_{V(t)} \varrho u d\tau \right] + \int_{\partial V} \varrho u (u_{\text{rel}} \cdot dA) + \int_{V(t)} \left( \frac{\partial p}{\partial r} + \varrho \frac{GM_r}{r^2} \right) d\tau = C_M, \quad (17)$$

$$\frac{d}{dt} \left[ \int_{V(t)} \varrho (e + \omega) d\tau \right] + \int_{\partial V} [\varrho (e + \omega) u_{\text{rel}} + j_w] \cdot dA + \int_{V(t)} p \operatorname{div} u d\tau = -C_E, \quad (18)$$

$$\frac{d}{dt} \left[ \int_{V(t)} E d\tau \right] + \int_{\partial V} [E u_{\text{rel}} + F] \cdot dA + \int_{V(t)} P \operatorname{div} u d\tau = C_E, \quad (19)$$

$$\frac{d}{dt} \left[ \int_{V(t)} \frac{F}{c^2} d\tau \right] + \int_{\partial V} \frac{F}{c^2} (u_{\text{rel}} \cdot dA) + \int_{V(t)} \left( \frac{\partial P}{\partial r} + \frac{F}{c^2} \frac{\partial u}{\partial r} \right) d\tau = -C_M, \quad (20)$$

$$\frac{d}{dt} \left[ \int_{V(t)} \varrho \omega d\tau \right] + \int_{\partial V} \varrho \omega u_{\text{rel}} \cdot dA = \int_{V(t)} (S_\omega - \tilde{S}_\omega - D_{\text{rad}}) d\tau, \quad (21)$$

$$C_M = \int_V \kappa \varrho \frac{F}{c} d\tau, \quad C_E = \int_V \kappa \varrho (4\pi S - cE) d\tau, \quad P = \frac{1}{3} E. \quad (22)$$

The equations connect *matter*, described by mass-density,  $\varrho$ , gravitating mass,  $M_r$  interior to radius,  $r$ , velocity,  $u$ , specific internal energy,  $e$ , gas pressure  $p$ , and *radiation*, characterised by radiation energy density per unit volume,  $E$ , radiative flux density per unit surface area  $F$ , and radiative pressure  $P$ , to convective eddies described by the specific turbulent kinetic energy density per unit mass  $\omega$ , that is the square of a mean convective velocity,  $u_c = \sqrt{2/3}\omega$  and the convective energy flux density per unit surface area  $j_w$ . The connection runs via the matter-radiation coupling terms for momentum exchange,  $C_M$ , energy exchange (absorption and emission of radiation),  $C_E$  and radiative cooling of convective elements,  $D_{\text{rad}}$ .  $\kappa$  is the frequency average of the mass extinction coefficient per unit mass and  $S$  the source function.  $S_\omega$  and  $\tilde{S}_\omega$  are the production- and dissipation-rates per unit volume, of turbulent kinetic energy,  $\omega$  due convective eddy generation by buoyancy forces and eddy dissipation by viscous forces.  $V$  is the time dependent volume under consideration — usually a shell in a celestial body —,  $\partial V$  its surface,  $d\tau$  and  $dA$  are volume and surface elements, and  $u_{\text{rel}}$  is the relative velocity of the flow across the volume surface. Wuchterl and Tscharnuter (2003) discuss how to set up and solve these equations and describe solutions relevant to the formation of stars and brown dwarfs. The authors calculate the collapse of cloud fragments with masses ranging from 0.05

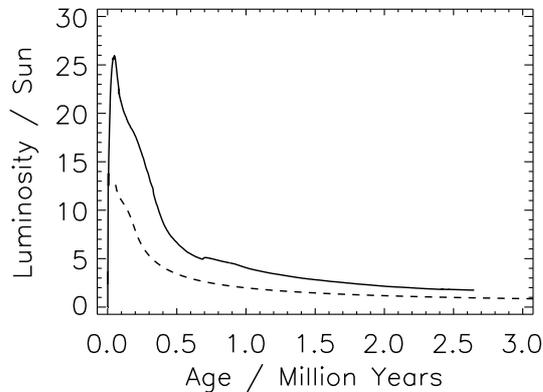


Figure 1: Early stellar evolution: collapse vs. Hayashi-line-contraction. Luminosity as function of age for a solar mass star. *Full line*: calculated from the protostellar collapse by Wuchterl and Tscharnuter (2003). *Dashed line*: the quasi-hydrostatic contraction of an initially fully convective young star by D’Antona and Mazzitelli (1994).

to  $10 M_{\odot}$  and discuss the consequences for the hydrostatic stellar evolution on the pre-main sequence.

## 7 Early stellar evolution — hydrostatic versus collapse

We now look at the key differences between hydrostatic and collapse calculations of early stellar evolution for the case of one solar mass. For the hydrostatic comparison case we follow Wuchterl and Tscharnuter (2003) and chose the calculations by D’Antona and Mazzitelli (1994), because atmospheric treatment, equations of state, opacities and convection treatment closely match those of Wuchterl and Tscharnuter (2003). Comparison to studies that include more physical processes (e.g., disc accretion or frequency dependent photospheric radiative transfer) can than be made by using existing intercomparisons of different hydrostatic studies to the D’Antona and Mazzitelli (1994) study. The luminosity as a function of age is shown for the collapse of a Bonnor-Ebert-sphere and a hydrostatic, contracting, initially fully convective young star in Fig. 1. Both studies use close to identical equations of state and calibrate mixing length theory of convection to the Sun. The two important differences are (1) the initial conditions and (2) the model equations.

**Initial conditions:** The starting point for collapse is a solar mass Bonnor-

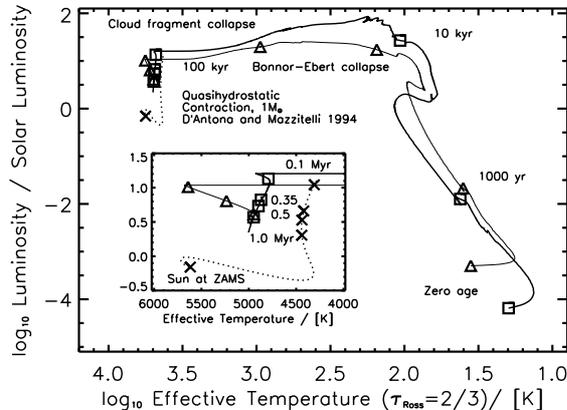


Figure 2: Protostellar collapse and early stellar evolution for a solar mass in a large theoretical Hertzsprung-Russel diagram. The collapse resulting from a fragmenting cloud (thick line) and a Bonnor-Ebert-sphere (thin line) are compared to a quasi-hydrostatic pre-main sequence calculation (D’Antona and Mazzitelli (1994)). After Wuchterl and Klessen (2001).

Ebert-sphere as the initial gravitationally unstable cloud fragment. The calculation of pre-main sequence contraction starts with an initially high luminosity, fully convective structure.

**Equations:** Collapse is calculated using convective radiation fluid-dynamics with an equation of motion and time-dependent radiative and convective energy transfer. The calculation of pre-main sequence contraction assumes hydrostatic equilibrium and accounts for time-dependence only due to slow (very subsonic) gravitational contraction via the energy equation. To a good approximation the equations of the contraction-calculation, D’Antona and Mazzitelli (1994) are a hydrostatic limiting case of the collapse equations by used Wuchterl and Tscharnuter (2003).

The collapse calculation starts at zero luminosity (at the beginning of cloud collapse) and stays above the luminosity of the quasi-hydrostatic contraction calculation to beyond 2.5 Ma. The cloud collapse does not lead to a fully convective structure as assumed for the hydrostatic calculation. Even after most of the mass is hydrostatic, most of the Deuterium has been burnt and accretion effects have ceased to dominate, at approximately 0.7 Ma, the internal structure stays partially radiative. The inner two thirds in radius remain radiative with a convective shell in the outer third of the radius — reminiscent of the present solar interior structure. Wuchterl and Tscharnuter (2003) found similar results from 2 down to 0.05  $M_{\text{Sun}}$  Bonnor-Ebert spheres, indicating that at least spherical collapse would not lead to fully convective structures over a consider-

able mass range. The question then was whether the initial cloud conditions or the cloud environment or non-spherical effects could change the result. Therefore Wuchterl and Klessen (2001)<sup>10</sup> studied the fragmentation of a large, dense molecular cloud by isothermal hydrodynamics in three dimensions and followed the collapse of one of the resulting fragments, that was closest to a solar mass, throughout the non-isothermal phases to the end of accretion using the spherically symmetric equations of Wuchterl and Tscharnuter (2003). Despite the very different cloud environment — interactions with neighbouring fragments, competitive accretion, varying accretion rate, and orders of magnitude higher average accretion rates — the structure of the resulting, young solar mass star after one Ma was almost identical to the one resulting from the quiet Bonnor-Ebert collapse.

There were large differences during the embedded, high luminosity phases and the earliest pre-main sequence phase (cf. Fig. 2) but at 1 Ma, when the accretion-effects had become minor the over-all structure resulting from the Bonnor-Ebert-collapse was confirmed: a convective shell on top of a radiative interior. This has consequences for the observables of young stars at young ages. A solar precursor at 1 Ma should have twice the luminosity and an effective temperature that is 500 K higher than the one of the respective, fully convective structure resulting from a hydrostatic, high luminosity star.

The differences of the young Sun properties at 1 Ma properties are the result of a number of differences that result when early stellar evolution is calculated directly from the cloud collapse instead of the hydrostatic evolution of initially high luminosity, fully convective structures. The collapse calculation predict a series of changes to the classical picture. In summary (Wuchterl and Tscharnuter (2003)):

1. young solar mass stars are not fully convective when they have first settled into hydrostatic equilibrium,
2. their interior structure with an outer convective shell and an inner radiative core extending across 2/3 of the radius rather resembles the present Sun than a fully convective structure,
3. in the Hertzsprung-Russel-diagram they do not appear on the Hayashi line but to the left of it,
4. most of their Deuterium is burned during the accretion phases,
5. Deuterium burning starts and proceeds off-centre, in a shell,
6. therefore there is no thermostatic effect of Deuterium during pre-main sequence contraction and hence no physical basis for the concept of the stellar birthline, as proposed by Stahler (1988a).
7. These results are independent of the accretion rates during the cloud fragmentation phase and non-spherical effects in the isothermal phase.

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<sup>10</sup>The Wuchterl and Tscharnuter (2003) article was submitted in 1999 and subject to 4 years of peer reviewing by numerous reviewers.

These differences have important consequences for surface abundances (of Deuterium and Lithium), rotational evolution, the stellar dynamo and stellar activity that have to be worked out.

The observational tests are still to be done. What is needed are very young binary systems where masses can be determined independent of the model calculations *and* where the stellar parameters, effective temperature, surface gravity, or ideally the radius can be determined with sufficient accuracy. Furthermore the binary has to be sufficiently wide to exclude interactions during accretion and the evolution to the observed stage.

But for theoretical reasons alone — namely the requirement of a physical description of star formation and protostellar collapse — the dynamical models should be used when masses of stars, brown dwarfs and planets are determined from luminosities, effective temperatures, gravities or radii of those objects. At least for the ages that can be currently covered with such models, i.e. up to 10 Ma.

## 8 Protoplanetary Disks

In the previous sections we have shown how the problem of star formation can be solved when angular momentum is neglected. The gravitational collapse of a cloud is then stopped when its central parts become opaque, heat up compressively and the thermal pressure finally re-balances gravity. Because pressure and gravity act isotropically the resulting structures, young stars, are spherical. When angular momentum is taken into account there is a second agent that can bring the collapse to a stop: inertia that gives rise to the centrifugal force. Even a slowly rotating cloud spins up dramatically as material collapses towards the centre reducing the axial distance by many orders of magnitude<sup>11</sup> under conservation of angular momentum. Unlike gas pressure, the centrifugal force is anisotropic and directed perpendicular to the axis of rotation. Hence collapse parallel to the axis of rotation is not modified by rotation. Gas can fall directly onto the central protostar along the polar axis and parallel to it towards the equatorial plane. The centrifugal force builds up during radial infall until it balances gravity at the *centrifugal radius*,

$$R_{\text{centrifugal}} = \frac{R^4 \omega^2}{GM}, \quad (23)$$

for a cloud of mass,  $M$ , initial radius  $R$ , rotating with an initial angular frequency  $\omega$ . Upon approach to that radius, the gas flow is more and more directed towards a collapse parallel to the axis of rotation. Finally material arrives in the equatorial plane. There, the vertical component of the central stars' gravitational force, i.e. the component parallel to the rotation axis, is zero. The radial component of the primary's gravitational force is balanced by the centrifugal forces. The cloud has collapsed to a flattened structure in the stellar

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<sup>11</sup>Typically by a factor of 1000 from 50 000 AU of the initial radius of a solar mass cloud fragment to 50 AU for the disk radius.

equatorial plane — a circumstellar disk. Because the gas in the disk has a finite pressure, the disk has a finite thickness determined by the force equilibrium of gas pressure and the vertical component of the stars gravity. That is another hydrostatic equilibrium, but only in one direction. The force balance in the radial direction is somewhat more subtle. Because the primary's gravity increases with decreasing orbital distance, parts of the disk, that are nearer to the stars are higher pressurised (more compressed and hence thinner in the vertical direction) than the one's further out. Therefore there is a radial pressure gradient in the disk that contributes to the force balance. Without that pressure, the force equilibrium would be gravity vs. centrifugal force, leading to the usual circular keplerian orbit. With the gas pressure, an orbiting blob of gas is subject to the outward directed pressure gradient of the disk. That partially reduces the effect of gravity. As a result the centrifugal force required for a balance is slightly less than in vacuum and the disk orbits somewhat below the respective Kepler-speeds. While this story is simply told, the details bring an enormous computational task so that there are only very few studies in the literature (e.g. Morfill et al. (1985), Tscharnuter (1987)) that can actually calculate the protoplanetary nebula as outcome of the collapse and follow the evolution from the cloud to stages where planet formation is expected. Again the problem is the non-isothermal part of the evolution complicated further by the incomplete knowledge about the disk angular momentum transfer processes.

If such processes are assumed the cloud collapse results in a central pressure-supported protostar and a centrifugally supported accretion disk (Tscharnuter (1987)) that may become a protoplanetary nebula.

We can only briefly list the most important elements of our knowledge or more appropriately our ignorance of quantitative protoplanetary disk structure here. For a discussion we refer to Wuchterl et al. (2000), Wuchterl (2004a) and with emphasis on Jupiter to Lunine et al. (2004). The key problem is that presently neither observation, due to resolution and sensitivity restrictions nor theory, due to computational difficulties and the problems with the angular momentum transfer process can provide sufficient information about the physical and chemical state of the protoplanetary nebula to build a planet formation theory on that. However, the following concepts and constraints have been collected for the properties of protoplanetary nebulae:

1. Theoretically protoplanetary nebulae may form as a by-product of the collapse of clouds with properties that are observed in actual clouds on the sky.
2. Accretion disks form when an appropriate anomalous viscosity exists, that allows accretion of mass onto the central protostar and outward angular momentum transfer through the discs. Candidate processes for the source of that viscosity are turbulence driven by either a sheer instability in the keplerian flow, a convective instability due a vertical temperature gradient in the disk, or a magneto-centrifugal instability in a conductive disk with a small seed magnetic field as well as gravitational torques induced by non-axisymmetric structures in the disk.

3. Assuming that km sized planetesimals form, and the material presently found in the solar system is converted into such planetesimals, the theory of planetesimal accretion by growth via pairwise collisions can show that the terrestrial planets form within about 100 Ma. A similar statement holds true for the cores of the giant planets, but it is necessary to assume that the original planetesimal disk in the outer solar system is a factor of a few more massive than the amount of condensible elements that is presently inferred for the outer solar system (mostly the condensible element cores of the giant planets). There are two problems for solar system formation theory here: planetesimal formation and the time-scales for giant planet formation, that will be discussed below.
4. From meteoritic chronology disk life-times can be inferred as time span when there was chemistry in the presence of nebula gas and impact driven melting during planetesimal accretion, Wadhwa and Russell (2000). The events can be absolutely dated from the decay of radio-nucleides (Lugmair and Shukolyukov (2001), Allègre et al. (1995)) and planet formation time-scales can be derived by relative chronology that points to a few hundred Ma. Recent studies obtain a formation time-scale of the Earth mantle off approximately 30–60 Ma, Yin et al. (2002) depending on the accretion scenario that determines the size and frequency of reservoirs that contribute to the mixing process that produces the isotopic evidence that can be analyzed today.
5. Observations in mm-wavelength have shown that the majority of young stars has circumstellar material of a few to ten percent of the stellar mass.
6. In many cases the inferred disks haven been resolved by mm-interferometry, optical and infrared imaging. Sizes range from disk radii of 1000 AU down to the resolution limits.
7. The properties in these disk are only weakly constrained by observation. Mostly because of insufficient resolution. mm-interferometry is able to provide information about the temperature, density and turbulent motion for scales down to 50 AU in the best cases, i.e. for bright, nearby systems.
8. The disk structure below 50 AU, is spatially unresolved. Spectral energy distributions can be used to infer properties via disk modelling. From near and mid-IR data, the surface-properties of disks can be inferred for distances of a few to a few ten stellar radii.
9. Those observations show a decay of IR-emission from inner disk regions on time-scales of a few to a few ten Ma. These observation give statistical information for the frequency of detected IR emission for clusters that can be dated via the stellar pre-main-sequence-evolution. Because a cluster sample is observed the obtained time-scale may indicate various evolutionary events in the systems (binaries) and their environment, with planet formation or disk dispersal being possible underlying processes.

In summary, the theoretical prediction of the frequent occurrence of circumstellar disks is well corroborated by observations. The observationally inferred disk masses are in the expected range of a few to ten percent of the stellar masses. Evolution of disk indicators is seen on the time-scales expected for planet formation. But the information about *local* disk properties that planet formation theory could use as input is presently beyond observational capabilities for orbital distances below 50 AU. That unfortunately correspond to the solar system size and even less accessible are the orbital distances of less than 5 AU of the extrasolar planets detected by radial velocity and transit techniques. In search for more information about the planet formation era, with hardly an alternative left, we now turn to our well studied home-system.

Our approach is to look at the solar system first and then try to generalise protoplanetary nebula structure by assuming that planet formation might occur in any gravitationally stable nebulae. Thus we will obtain a range of plausible protoplanetary nebula disks that are scaled from what we know from reconstructions of the solar nebula from the distribution of matter in the present solar system.

## 9 The Solar System

More planetary candidates are now known in our galactic neighbourhood than planets orbit our star. But the Solar System is by far the best studied and most completely known system. Masses, radii and composition of the major and some minor bodies are known. Almost a million orbital elements allow a detailed study of the dynamical and stability properties of the system, Lecar et al. (2001). The information about the Sun, the planets and their satellite systems provides a clear picture of the angular momentum distribution. Interior structures and surface properties contain a record of the formation history. The heat budgets of the giant planets show an excess of emitted radiation over absorbed sunlight (except for Uranus where only an upper limit is available). These faint intrinsic luminosities, in the  $nL_{\odot}$  range ( $1nL_{\odot} = 10^{-9}$  solar luminosities) are directly related to the formation process that stored the heat 4.5 Ga ago (1 Ga =  $10^9$  years). The solid, icy and rocky surfaces of terrestrial planets and many satellites show a record of impacts that samples a time-range reaching from the present, e.g. the surface of Io, back to the formation epoch, e.g. in the lunar highlands. Radioactive dating provides absolute ages for the Earth, Moon and meteorites. The latter provide an accurate age of  $4.565 \pm 1$  Ga for the oldest nebula condensates, the Calcium-Aluminium rich inclusions in primitive meteorites, most prominently, Allende, Allègre et al. (1995). Ongoing processes as orbital evolution, asteroidal collisions and even large impacts, as those of the fragments of comet Shoemaker-Levy 9 onto Jupiter can be studied directly and in great detail. Finally there is abundant information gathered by fly-by space-craft missions as the Voyagers, orbiters as Galileo, Mars-Odyssey and Mars-Express, as well as from in-situ exploration by landers and atmospheric entry probes for the Moon, Venus, Mars, Jupiter and last but not least

the successful atmospheric entry and landing of the *Huyghens* probe on Saturn's moon Titan in January 2005 and finally we are looking forward to the artificial collision with a comet of the impactor of the *Deep-Impact* mission that is scheduled for 4th July 2005 to bring material from the interior of a comet to light.

Our four giant planets contain 99.5% of the angular momentum of the Solar System, but only 0.13% of its mass. Terrestrial planets contribute another 0.16%, to the angular momentum. On the other hand, more than 99.5% of the mass and thermal energy of the planetary system is in the four largest bodies, with the remaining 0.5 % mostly in the second group-of-four, the terrestrial planets. Modern models of the interiors and evolution of giant planets in our Solar System account for the high pressure properties of hydrogen, helium and the heavier elements as well as energy transfer by radiation and convection. When fit to the observed global properties of Jupiter at an age of 4.5 Ga they show that 10–42 of the 318 Earth-masses are due to heavy elements. That corresponds to 3 to 13 %. The respective mass fractions implied for the other solar system planets are all higher. That points to a bulk enrichment of heavy elements more than a factor of two above solar composition and implies heavy element cores ranging from greater than one Earth mass to a considerable fraction of the total mass (Guillot (2005, 1999), Wuchterl et al. (2000)). The heavy element enrichment is even more obvious in terrestrial planets that may also be viewed as cores of failed giant planets. Hydrogen and helium that constitute for 98% of the Sun's mass and between 87% and 15% of the giants is a minor constituent of terrestrial planets. Forming planets enriches heavy elements relative to the central star, that is formed from the same protostellar cloud. Such an extensive enrichment is not predicted by any mechanism proposed for the formation of stars and brown dwarfs. That alone is already indicating that planet formation is fundamentally different from star formation.

## 10 Solar System Formation

The distribution of mass and angular momentum in the Solar System can be understood on the basis of the *nebula hypothesis* Kant (1755). The nebula hypothesis assumes concurrent formation of a planetary system and a star from a centrifugally supported flattened disk of gas and dust with a pressure supported, central condensation (Laplace (1796), Safronov (1969), Lissauer (1993)). Flattened preplanetary nebula disks explain the coplanarity and circularity of planetary orbits by the respective properties of the parent disk. Theoretical models of the collapse of slowly rotating molecular cloud cores have demonstrated that such preplanetary nebulae are the consequence of the observed cloud core conditions and the dynamics of radiating fluids, provided there is a macroscopic angular momentum transfer process, Morfill et al. (1985). Assuming turbulent viscosity to be that process, dynamical models have shown how mass and angular momentum separate by accretion through a viscous disk onto a growing central protostar (Tscharnuter (1987), Tscharnuter and Boss (1993)).

Such cloud collapse calculations, however, still do not reach to the evolutionary state of the nebula where planet formation is expected. Observationally inferred disk sizes and masses are overlapping theoretical expectations and confirm the nebula hypothesis. High resolution observations at millimeter-wavelengths are sensitive to disk conditions at orbital distances  $> 50$  AU. However, observations thus far provide little information about the physical conditions in the respective nebulae on scales of 1 to 40 AU, where planet formation is expected to occur. Planet formation studies therefore obtain plausible values for disk conditions from nebulae that are reconstructed from the present planetary system and disk physics. The so obtained *minimum reconstituted nebula masses* defined as

the total mass of solar composition material needed to provide the observed planetary/satellite masses and compositions by condensation and accumulation,

are a few percent of the central body for the solar nebula *and* the circumplanetary protosatellite nebulae (Kusaka et al. (1970), Hayashi (1980), Stevenson (1982)). See Lunine et al. (2004) for a more detailed discussion of reconstructing the preplanetary nebula from observational constraints provided by solar system data.

## 11 Planet Formation — the Problem

Giant planet formation requires (1) a compression of the solar nebula gas by about 10 orders of magnitude to form a gaseous condensation held together by its own gravity, at Jupiter’s present mean density of  $1.33 \cdot 10^3 \text{ kg/m}^3$ , and (2) an enrichment of the heavy elements — that are condensible in the nebula — by typically a factor of at least three above the nebula value, most likely with a substantial fraction contained in a core. Gas in the midplane of a *minimum mass solar* nebula typically has a density of  $10^{-8} \text{ kg/m}^3$  at Jupiter’s present orbital radius (Hayashi et al. (1985)) and a temperature around 100 K. The nebula gas pressure, the young Sun’s tides and the radially decreasing orbital velocities in a circumstellar disk, that shows an almost ‘keplerian’ shear, counteract the compressing force of nebula-gas self-gravity. Accordingly most circumstellar nebulae — modelled and observed — are gravitationally stable. Unlike in interstellar clouds, larger mass fragments in a circumstellar disk are generally not more unstable. That is because larger mass fragments at given nebula densities require larger scales that are subject to stabilisation against self-gravity by the stellar tidal pull and the keplerian shear. While in an interstellar cloud sufficiently large scales are always gravitationally unstable, nebulae are stabilised on short *and* long scales.

In addition to the above mechanical barriers against gravitational self-compression of nebula gas, there is a thermal barrier: a Jupiter mass fragment is optically thick even under unperturbed nebula conditions. The optical depth,  $\tau$ , for a

blob of mass  $M$ , at nebula densities,  $\rho_{\text{neb}}$ , and for typical (dust) opacity,  $\kappa$  is:

$$\tau = 36 \left( \frac{M}{[M_{\text{Jupiter}}]} \right)^{1/3} \left( \frac{\kappa}{[0.01 \text{ m}^2/\text{kg}]} \right) \left( \frac{\rho_{\text{neb}}}{[10^{-8} \text{ kg/m}^3]} \right)^{2/3}, \quad (24)$$

were the problem oriented units in the scaling have been chosen according to the values for the Jupiter position in Hayashi's minimum mass nebula. Any rapid, i.e. dynamical compression under such conditions will result in a temperature-increase determined by the efficiency of transfer processes and a much stronger counteracting pressure than in a simple isothermal analogue of protostellar collapse scaled down to planetary masses. In the stellar case with  $\tau \ll 1$ , compressional heat leaks out as fast as it is produced keeping the parent cloud isothermal for many<sup>12</sup> orders of magnitude in compression. The rapid compression under optically thick conditions in the planetary case produces an immediate thermal pressure increase, that typically leads to a slow down of compression from the dynamical time-scale of the fragment, a few years, to the thermal (cooling) timescale which is found to be of the order of million years in detailed models. Compression of the nebula gas — and therefore giant planet-formation, their mass-growth and evolution — is then controlled by the heat loss of the fragment or protoplanet, e.g. Safronov and Ruskol (1982). Collapse, i.e. fast, gravity-driven compression in an essentially free-falling manner, with pressure playing a negligible role is a very unlikely event under such conditions.

The physical nature of giant planet formation — collapse, thermally controlled quasi-hydrostatic contraction or static accumulation — is decided by the dynamical stability of the nebula and the pressure build-up inside the protoplanets regulated by the thermal budget of the protoplanetary envelopes. The thermal budget contains (1) heating due to contraction of the gaseous envelopes, (2) dissipation of planetesimal kinetic energy at impacts, and, (3), 'cooling' due to energy transfer to the ambient nebula by radiation and convection.

## 12 How to Compress by $10^{10}$ ?

The transition from dilute, weakly gravitating nebula conditions to compact planets with a spherical shape rounded by self-gravitation involves a compression of ten orders of magnitude. The nebula gas apparently had to be compressed by a macroscopic process from the earliest stages of planetary growth to the final planetary densities of  $\sim 1000 \text{ kg/m}^3$ . For stars, the runaway of the Jeans-instability easily multiplied the factor  $10^{10}$  to the original cloud density, only inhibited to proceed further by the centrifugal force forming the very nebula. But unlike the protostellar collapse the planetary compression process cannot be analogous for the remaining  $10^{10}$  from the nebula to the final planetary densities because it has to enrich the condensible material at the same time.

Since the 1970s two hypothesis have been discussed that try to account for nebula gas compression *and* condensible element enrichment. The *gravitational*

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<sup>12</sup>Factor  $\sim 10^{10}$  in density for the collapse of a Jeans-critical solar mass.

*disk instability hypothesis* tries to find a nebula-analogue of the gravitational ‘Jeans’-instability of star formation and *the nucleated instability hypothesis* aims to explain giant planet formation as the consequence of the formation of solid, condensable element planetary embryos that act as gravitational seeds for nebula gas capture/condensation. The disk-instability hypothesis requires nebulae that undergo self-compression in a dynamically unstable situation and lead to a transition from a smooth regular disk to an ensemble of clumps in orbit around the Sun. Such clumps may be regarded as candidate precursors of protoplanets.

The nucleated instability model looks at giant planet formation as a second step of rocky planet formation. A terrestrial planet embryo acts as a gravitating seed to permanently bind nebula gas thus forming a massive gaseous envelope around a condensible element core.

The key problem for both compression processes is that protoplanetary disks are only weakly self-gravitating equilibrium structures supported by centrifugal forces augmented by gas pressure.<sup>13</sup> Any isolated, orbiting object below the Roche density is pulled apart by the stellar tides. Typical nebula densities are more than two orders of magnitude below the Roche density, so a finite nebula pressure is needed to confine a condensation of mass  $M$  inside its tidal or Hill-radius at orbital distance  $a$ :

$$R_T = a \left( \frac{M}{3M_\odot} \right)^{1/3}. \quad (25)$$

Mature planets are dense enough so that their radii are much smaller than the Hill-radius. Hence their high densities, and, as a consequence, their high surface gravities usually protect them from tidal disruption or noticeable mass-loss. Stellar companions for comparison reduce their densities due to evolutionary effects when they become giants. Consequently their radii may approach the Hill values depending on their orbital radii. The consequence is Roche-lobe overflow when such stars are very close in binary systems. Planet formation requires a somewhat inverse process, where an extra force compresses the nebula material into the Hill-sphere, allowing more material to flow into the Roche lobe and to increase the planetary mass inside the lobe. All theories of planet formation rely on an extra gravity field to perform this compression.

### 13 How to Provide the Extra Gravity Field?

Giant planet formation theories may be classified by how they provide the gravity enhancement:

1. the *nucleated instability* model relies on the extra gravity field of a sufficiently large solid core (condensed material represents a gain of ten orders of magnitude in density and therefore self-gravity compared to the nebula gas),

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<sup>13</sup>For a review of circumstellar and protoplanetary disk I recommend the review chapters in the *Protostars and Planets IV* book, Hollenbach et al. (2000); Calvet et al. (2000); Beckwith et al. (2000).

2. a *disk instability* may operate on length scales between short scale pressure support and long scale tidal support, or
3. an *external perturber* could compress an otherwise stable disk on its local dynamical time scales, e.g., by accretion of a clump onto the disk or rendezvous with a stellar companion.

## 14 From Dust to Planets

Dust growth in the nebula via pairwise collision to cm sizes is now fairly well understood theoretically and experimentally (see Lissauer (1993)). A key open question is how the transition from  $\sim 0.1$  m dust-agglomerates to km sized planetesimals can be accomplished. Planetesimals, that are the building blocks of gravitationally controlled planetary accumulation, may form by a gravitational instability of a dust subdisk or by continued growth via pairwise collisions provided growth is sufficiently large to dominate over losses due to a radial inward drift (cf. Lissauer (1993)). I will follow the *planetesimal hypothesis* here without further discussion and assume that protoplanetary nebulae are forming km sized bodies made of condensible elements within a time-frame of about 10000 a, see e.g. Hueso and Guillot (2003).

The next step, *runaway planetesimal accretion*, proceeds from  $\sim 10^{-9} M_{\text{Earth}}$  planetesimals to  $\sim 0.1 M_{\text{Earth}}$  *planetary embryos*. In the simplest case a single planetesimal grows within a swarm of other planetesimals of density  $\rho_{\text{sw}}$ . For a relative velocity,  $v$  the planetesimals mass,  $M$  grows according to the *particle in box accretion-rate*:

$$\frac{dM}{dt} = \rho_{\text{sw}} v \pi R_s^2 \left[ 1 + \left( \frac{v_e}{v} \right)^2 \right], \quad (26)$$

where  $R_s$  is the distance between the planetesimal centres at contact, an  $v_e$  the escape speed at contact. Assuming that the mean ratio of horizontal to vertical motions remains fixed the rate can be rewritten using the surface mass density  $\Sigma_{\text{solid}}$  of the planetesimal swarm:

$$\frac{dM}{dt} = \frac{\sqrt{3}}{2} \Sigma_{\text{sw}} \Omega_{\text{Kepler}} \pi R_s^2 F_g, \quad (27)$$

where we also introduced the gravitational focusing factor,  $F_g = 1 + (v_e/v)^2$ , and used the Keplerian angular velocity  $\Omega_{\text{Kepler}} = \sqrt{GM/r^3}$ . Half the ratio of relative speed to escape speed is also known as the Safronov Number,  $\theta = 1/2(v_e/v)^2$ . Depending on the planetesimal swarm properties the gravitational focusing factor is typically a few 1000 during early runaway growth, and  $< 8$  during the late stages of planetary accretion, see Lissauer (1993).

Based on detailed n-body calculations of a number of planetary embryos together with the self-consistent determination of the properties of the planetesimal swarm, Tanaka and Ida (1999) estimate the respective runaway accretion-

time for protoplanets of mass  $M_p$ , at orbital radius  $a$ :

$$\frac{\tau_{\text{grow}}}{[\text{a}]} = 8 \cdot 10^5 \left( \frac{M_p}{M_{\text{Earth}}} \right)^{1/3} \left( \frac{a}{[\text{AU}]} \right)^{12/13}. \quad (28)$$

Mutual interactions and accretion of planetesimals and embryos are accounted for. Runaway accretion stops at the *isolation mass*. The isolation mass is reached when a planetary embryo has accreted all planetesimals within its gravitational range — the so called *feeding-zone*. The feeding zone of an embryo extends typically a few Hill-radii,  $\sim 5$ , say, around its orbit. The values of the isolation mass depend on the initial nebula solid surface density, that specifies the amount of condensible material that is available per unit nebula surface area, and the orbital radius of the embryo. Values for the isolation mass in a minimum mass nebula are typically,  $1 M_{\text{Earth}}$  in the outer solar system and a Mars-mass,  $0.1 M_{\text{Earth}}$ , at 1 AU. Protoplanets with masses larger than the isolation mass then must enter an *oligarchic growth stage* and have much larger growth times. Kokubo and Ida (2002) estimated the total accretion times of planetary cores through runaway accretion and the late phases of oligarchic growth in the jovian planet region to be:

$$\frac{T_{\text{grow}}}{[\text{a}]} \sim 9 \cdot 10^4 \left( \frac{e}{h_M} \right)^2 \left( \frac{M}{[10^{26} \text{ g}]} \right)^{1/3} \left( \frac{\Sigma}{[4 \text{ g/cm}^2]} \right)^{-1} \left( \frac{a}{[5 \text{ AU}]} \right)^{1/2}. \quad (29)$$

For an eccentricity in Hill-units,  $e/h_M$ ,  $h_M \equiv R_{\text{Hill}}$ , solid surface density,  $\Sigma$ , final proto-planetary mass,  $M$  and semi-major axis,  $a$ . They estimate that the final condensible element mass of a protoplanet at 5 AU would be  $5 M_{\text{Earth}}$ . Accretion would be completed in 40 Ma. A  $9 M_{\text{Earth}}$  core at Saturn's position would require 300 Ma.

These growth times likely have to be shortened due to the enhancement of collision cross-sections of planetary embryos by their envelopes, Inaba and Ikoma (2003). The effect is especially strong when it is considered that some collisions of smaller embryos lead to fragments that are more affected by gas drag in the atmospheres of the larger embryos. Inaba et al. (2003) find that their largest planetary embryo at 5.2 AU, with a mass of  $21 M_{\text{Earth}}$ , formed in 3.8 Ma!

## 15 Solar System Formation Modelling

Understanding the formation of the solar system presently means the reconstruction of a history. That approach is necessary because, due to incomplete knowledge of important physical processes, it is necessary to include parameterised descriptions of uncertainties. The most famous parameterisations is the one of anomalous, turbulent  $\alpha$ -viscosity that is assumed to allow angular momentum redistribution and accretion of mass onto the star. Next an initial structure model of the pre-planetary nebula has to be assumed. Two mayor classes of nebula models may be distinguished: (1) *active* viscous  $\alpha$ -disks (e.g.

Ruden and Pollack (1991), Drouart et al. (1999), Hueso and Guillot (2003) and (2) *passive* disks that are heated by absorbed stellar radiation (e.g. the *Kyoto* minimum mass nebula, Hayashi et al. (1985)). Once the class and parametrisation of the nebula model is chosen (passive or active) the planet formation processes have to be specified and parameterised. The key nebula processes and parameters are:

1. distribution of temperature and density as a function of orbital radius. That follows for a given class of nebula models from a chosen mass and mass distribution. In practice the discussion is parameterised by the local surface densities  $\Sigma_{\text{dust}}$  and  $\Sigma_{\text{gas}}$  of nebula condensates and nebula gas, respectively.
2. planetesimal properties and size distribution;
3. planetesimal collision properties, i.e. coefficients of restitution and outcome of collisions — merging into a larger planetesimal or fragmentation into smaller pieces;
4. energy transfer properties of the nebula gas:
  - (a) dust properties (size distribution, composition, mineralogy) to determine the dust opacities and the efficiency of radiative transfer. Nebula dust differs considerably in size and composition from the interstellar dust due to growth and condensation processes in the nebula;
  - (b) a prescription and parametrisation for convective energy transfer.

Solar system data are used at two stages: (1) in the construction of the nebula surface densities and (2) in the adjustment of parameters by comparing the final outcome of planets to the empirical data from solar system planets. Because the uncertainties in the initial nebula structure are very large, the respective structure-parameters are the prime ones that are adjusted. A typical procedure is as follows: In a first step a nebula is constructed e.g. by assigning a volume to every solar system planet. Then hydrogen and helium is added until the presolar abundances are reached. The resulting mass of solids and gas is smeared out across the volume and fitted to the chosen class of nebula models. The result is e.g. a minimum reconstituted nebula with solid and gas surface densities described by parameterised power laws (e.g. Hayashi et al. (1985)). For the so constructed nebula the outcome of planet formation is deduced in a multi-step process: (1) planetesimal formation, (2) planetesimal accretion, (3) formation of planetary envelopes, (4) nebula gas capture by large envelopes (5) termination of planetary accretion and dissipation of remnant nebula gas. A typical result for the minimum mass nebula is that predicted accretion times turn out to be much larger than plausible nebula life-times. In consequence the original assumptions going into the construction of the nebula are reconsidered. Lissauer (1987), e.g. described how a solid surface density increased by a factor less than ten could account for a Jovian planet within the time constraints. Wuchterl

(1993) showed how an increase of the gas surface density by less than a factor of ten would lead to a new class of protoplanets with massive envelopes, that dynamically could grow to a few hundred Earth masses (Wuchterl (1995a)). Pollack et al. (1996) adjusted nebula and planetesimal parameters to account for the accretion of Jupiter and Saturn with detailed models of planetesimal accretion and gaseous envelope capture. When coupled to evolutionary models, Guillot (1999, 2005). the observed properties of gravitational fields, the radii and present excess luminosities can be reproduced when interior structures are fitted by detailed planetary structure and evolution models with three compositional layers.

## 15.1 Gaseous Envelopes – Giant Planets

Planetesimals in the solar nebula are small bodies surrounded by gas. A rarefied equilibrium atmosphere forms around such objects. The question is then how massive the planetesimal or planetary embryo has to become to capture large amounts of gas. In particular at what mass it could bind more gas from the nebula than its own mass. The it could become a Jupiter or Saturn precursor-object. A *proto giant planet* would then form. The respective mass-values are referred to as the *critical mass*. Some care has to be taken because the usage of the term is not homogeneous in the literatures and physically differing variants are often used synonymously.

Because planetary masses are optically thick in the nebula (cf. Equ. 24), such objects are much hotter in the interior than the ambient nebula. Consequently, the energy budget of the envelope has been modelled more and more carefully. Mizuno (1980) calculated the first realistic protoplanetary structures that could be related to the solar system planets. Mizuno found, that anywhere in the nebula the required mass for gas capture would be similar. That could explain the similarity of the solar-system giant’s cores, despite their widely differing envelope and total masses. Bodenheimer and Pollack (1986)<sup>14</sup> accounted for heat generated by gravitational contraction of the envelopes by building quasi-hydrostatic models. Pollack et al. (1996) that planetesimal accretion would control the timing and onset of envelope accretion. Dynamic effects and possible accretion flows were added by Wuchterl (1989, 1990, 1991a,b). The hydrodynamical calculations showed that accretion was not the only pathway of planetary evolution und envelope ejection, hence mass loss can also occur at the critical mass. Wuchterl (1993) showed that protoplanetary structure and hence the critical mass can vary a lot once the outer envelopes become convective and Wuchterl (1995a) showed that largely convective protoplanets would allow the onset of accretion at much lower core masses than the dominating population of protoplanets with radiative outer parts. For further discussion see Wuchterl et al. (2000).

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<sup>14</sup>The group is continuing to refine their models in the quasi-static approximation. Pollack et al. (1996) included detailed planetesimal accretion rates, Bodenheimer et al. (2000) applied it to extrasolar planets and ? accounted for dust depletion effects due to planetesimal growth.

Most aspects of early envelope growth, up to  $\sim 10 M_{\text{Earth}}$ , can be understood on the basis of a simplified analytical model given by Stevenson (1982) for a protoplanet with constant opacity  $\kappa_0$ , core-mass accretion-rate  $\dot{M}_{\text{core}}$ , core-density  $\rho_{\text{core}}$ , inside the Hill-radius,  $r_{\text{Hill}}$ . The key properties of Stevenson’s model come from the ‘radiative zero solution’ for spherical protoplanets with static, fully radiative envelopes, i.e., in hydrostatic and thermal equilibrium. Wuchterl et al. (2000) presented an extended solution relevant to the structure of an envelope in the gravitational potential of a constant mass, for zero external temperature and pressure and using a generalized opacity law of the form  $\kappa = \kappa_0 P^a T^b$ .

The critical mass, defined as the largest mass a core can grow to with the envelope kept static is then given by:

$$M_{\text{core}}^{\text{crit}} = \left[ \frac{3^3}{4^4} \left( \frac{R_{\text{gas}}}{\mu} \right)^4 \frac{1}{4\pi G} \frac{4-b}{1+a} \frac{3\kappa_0}{\pi\sigma} \left( \frac{4\pi}{3} \rho_{\text{core}} \right)^{\frac{1}{3}} \frac{\dot{M}_{\text{core}}}{\ln(r_{\text{Hill}}/r_{\text{core}})} \right]^{\frac{3}{7}}, \quad (4)$$

and  $M_{\text{core}}^{\text{crit}}/M_{\text{tot}}^{\text{crit}} = 3/4$ ;  $R_{\text{gas}}$ ,  $G$ ,  $\sigma$  denote the gas constant, the gravitational constant, and the Stefan-Boltzmann constant respectively. The critical mass does neither depend on the midplane density  $\varrho_{\text{neb}}$ , nor on the temperature  $T_{\text{Neb}}$  of the nebula in which the core is embedded. The outer radius,  $r_{\text{Hill}}$ , enters only logarithmically weak. The strong dependence of the analytic solution on molecular weight  $\mu$ , led Stevenson (1984) to propose ‘superganymedeian puffballs’ with atmospheres assumed to be enriched in heavy elements and a resulting low critical mass as a way to form giant planets rapidly (see also Lissauer et al. (1995)). Except for the weak dependencies discussed above, a proto-giant planet essentially has the same global properties for a given core wherever it is embedded in a nebula. Even the dependence on  $\dot{M}_{\text{core}}$  is relatively weak: Detailed radiative/convective envelope models show that a variation of a factor of 100 in  $\dot{M}_{\text{core}}$  leads only to a 2.6 variation in the critical core mass Wuchterl (1995a).

However, other static solutions are found for protoplanets with *convective* outer envelope, which occur for somewhat larger midplane densities than in minimum mass nebulae (Wuchterl (1993), Ikoma et al. (2001)). These largely convective proto-giant planets have larger envelopes for a given core and a reduced critical core mass. Their properties can be illustrated by a simplified analytical solution for fully convective, adiabatic envelopes with constant first adiabatic exponent,  $\Gamma_1$ :

$$M_{\text{core}}^{\text{crit}} = \frac{1}{\sqrt{4\pi}} \frac{\sqrt{\Gamma_1 - \frac{4}{3}}}{(\Gamma_1 - 1)^2} \left( \frac{\Gamma_1}{G} \frac{\mathcal{R}}{\mu} \right)^{\frac{3}{2}} T_{\text{neb}}^{\frac{3}{2}} \rho_{\text{Neb}}^{-\frac{1}{2}} \quad (30)$$

and  $M_{\text{core}}^{\text{crit}}/M_{\text{tot}}^{\text{crit}} = 2/3$ . In this case, the critical mass depends on the nebula gas properties and therefore the location in the nebula, but it is independent of the core accretion-rate. Of course, both the radiative zero and fully convective solutions are approximate because they only roughly estimate envelope gravity and all detailed calculations show radiative *and* convective regions in proto-giant planets. The critical mass can be as low as  $1 M_{\text{Earth}}$ , and subcritical static

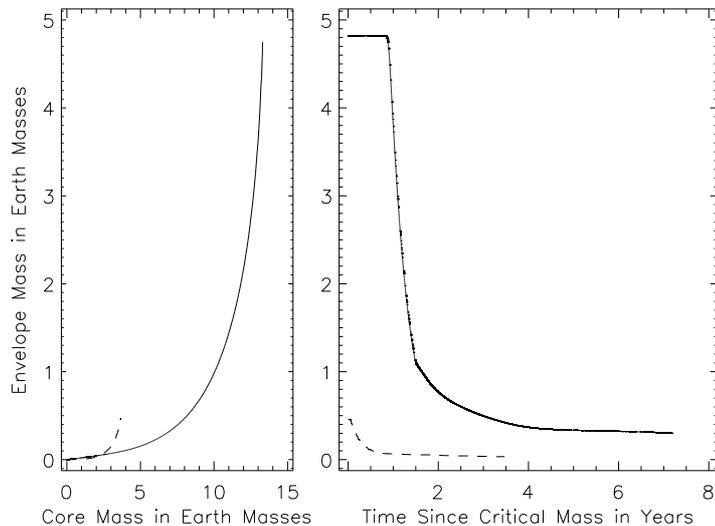


Figure 3: Hydrostatic envelope accretion due to a core growing by accretion of planetesimals (left) and hydrodynamical ejection of protoplanetary envelope gas due to pulsation-driven mass-loss (right). The evolution is shown for Mizuno (1980)’s Neptune conditions and a planetesimal accretion-rate of the core  $\dot{M}_{\text{core}} = 10^{-6} M_{\text{Earth}} \text{ a}^{-1}$  (full line). The dashed line is the same but with time-dependent MLT-convection, updated molecular opacities, and a particle-in-a-box core-accretion-rate, with a planetesimal surface density of  $10 \text{ kg m}^{-2}$  and a gravitational focusing factor of 2000, Wuchterl (1997).

envelopes can grow to  $48 M_{\text{Earth}}$ . See Wuchterl et al. (2000) and Wuchterl (1993) for more details. Ikoma et al. (2001) study largely convective protoplanets for a wide range of nebula conditions and show the limiting role of gravitational instability.

The early phases of giant planet formation discussed above are dominated by the growth of the core. The envelopes adjust much faster to the changing size and gravity of the core than the core grows. As a result the envelopes of proto-giant planets remain very close to static and in equilibrium below the critical mass (Mizuno (1980), Wuchterl (1993)). This has to change when the envelopes become more massive and cannot re-equilibrate as fast as the cores grow. The nucleated instability was assumed to set in at the critical mass, originally as a hydrodynamic instability analogously to the Jeans instability. With the recognition that energy losses from the proto-giant planet envelopes control the further accretion of gas, it followed that quasi-hydrostatic contraction of the envelopes would play a key role.

## 15.2 Hydrodynamic Accretion Beyond the Critical Mass

Static and quasi-hydrostatic models rely on the assumption that gas accretion from the nebula onto the core is very subsonic, and the inertia of the gas and dynamical effects as dissipation of kinetic energy do not play a role. To check whether hydrostatic equilibrium is achieved and whether it holds, especially beyond the critical mass, hydrodynamical investigations are necessary. Two types of hydrodynamical investigations of protoplanetary structure have been undertaken in the last decade: (1) linear adiabatic dynamical stability analysis of envelopes evolving quasi-hydrostatically Tajima and Nakagawa (1997) and (2) nonlinear, convective radiation hydrodynamical calculations of core-envelope proto-giant planets (see Wuchterl et al. (2000)). In the linear studies it was found that the hydrostatic equilibrium is stable in the case they investigated. The nonlinear dynamical studies follow the evolution of a proto-giant planet without a priori assuming hydrostatic equilibrium and they *determine* whether envelopes are hydrostatic, pulsate or collapse, and at which rates mass flows onto the planet assuming the mass is available in the planet's feeding zone. Hydrodynamical calculations that determine the flow from the nebula into the protoplanet's feeding zone are discussed in Sect. 23. The first hydrodynamical calculation of the nucleated instability, Wuchterl (1989, 1991a,b) started at the static critical mass and brought a surprise: Instead of collapsing, the proto-giant planet envelope started to pulsate after a very short contraction phase; see Wuchterl (1990) for a simple discussion of the driving  $\kappa$ -mechanism. The pulsations of the inner protoplanetary envelope expanded the outer envelope and the outward travelling waves caused by the pulsations resulted in a mass-loss from the envelope into the nebula. The process can be described as a pulsation-driven wind. After a large fraction of the envelope mass has been pushed back into the nebula, the dynamical activity fades and a new quasi-equilibrium state is found that resembles Uranus and Neptune in core and envelope mass (cf. Fig. 3, full line).

The main question concerning the hydrodynamics was then to ask for conditions that allow gas accretion, i.e., damp envelope pulsations. Wuchterl (1993) derived conditions for the breakdown of the radiative zero solution by determining nebulae conditions that would make the outer envelope of a 'radiative' critical mass proto-giant planet convectively unstable. The resulting criterion gives a minimum nebula density that is necessary for a convective outer envelope:

$$\frac{\rho_{\text{neb}}^{\text{crit}}}{[10^{-10} \text{g cm}^{-3}]} = 2.2 \left( \frac{T}{[100\text{K}]} \right)^3 \left( \frac{\nabla_s}{[2/7]} \right) \left( \frac{\mu}{[2.2]} \right) \left( \frac{\kappa}{[\text{cm}^2 \text{g}^{-1}]} \right)^{-1} \\ \left( \frac{\dot{M}_{\text{core}}}{[10^{-6} \text{M}_{\text{Earth}} \text{a}^{-1}]} \right)^{-1} \left( \frac{M_{\text{core}}}{[10 \text{M}_{\text{Earth}}]} \right)^{\frac{1}{3}} \left( \frac{\rho_{\text{core}}}{[5.5]} \right)^{-\frac{1}{3}} \quad (31)$$

Protoplanets that grow under nebula conditions above that density have larger envelopes for a given core and a reduced critical mass. For sufficiently large nebula densities Wuchterl (1995a) found that the pulsations were damped and

rapid accretion of gas set in and proceeded to  $300 M_{\text{Earth}}$ . The critical core masses required for the formation of this class of proto-giant planets are significantly smaller than for the Uranus/Neptune-type (see Wuchterl (1993, 1995a), Ikoma et al. (2001)).

## 16 Importance of Convection

Convection plays an important role in determining the mass of protoplanets by controlling energy transfer in the outer layers under specific nebula conditions. It also controls the dynamical behaviour of the protoplanetary envelopes beyond the critical mass as described in the last section. Most giant planet formation studies use zero entropy gradient convection, i.e., set the temperature gradient to the adiabatic value in convectively unstable layers of the envelope or use the time-independent mixing length theory. That is done for simplicity but can be inaccurate, especially when the evolution is rapid and hydrodynamical waves are present, Wuchterl (1991b). Furthermore convection in the outer layers of a protoplanet occurs under weak gravities and relatively low optical depths. Hence departures from an adiabatic behaviour might be expected. It was, therefore, important to develop a time-dependent theory of convection that can be solved together with the equations of radiation hydrodynamics in the entire protoplanetary flow-regime. Such a time-dependent convection model (Kuhfuß (1987)) has been reformulated for self-adaptive grid radiation hydrodynamics, Wuchterl (1995b) and applied to giant planet formation (Götz (1989), Wuchterl (1996, 1997)). In a reformulation by Wuchterl and Feuchtinger (1998), it closely approximates standard mixing length theory in a static local limit and accurately describes the solar convection zone and RR-Lyrae light-curves.

The heart of this convection model is a dynamical equation for the specific kinetic energy density,  $\omega$ , of convective elements. An equation accounts for creation of eddies by buoyancy, dissipation of eddies due to viscous effects as well as eddy advection and radiative losses:

$$\frac{d}{dt} \left[ \int_{V(t)} \varrho \omega d\tau \right] + \int_{\partial V} \varrho \omega u_{\text{rel}} \cdot dA = \int_{V(t)} \left( S_{\omega} - \tilde{S}_{\omega} - D_{\text{rad}} \right) d\tau \quad (32)$$

where the eddy kinetic energy generation-rate, the eddy dissipation-rate, the convective enthalpy flux, the reciprocal value of the mixing-length,  $\Lambda$  and the

time-scale for radiative eddy losses, respectively, are:

$$S_\omega = -\nabla_s \frac{T}{P} \frac{\partial P}{\partial r} \Pi \quad (33)$$

$$\tilde{S}_\omega = \frac{c_D}{\Lambda} \omega^{3/2} \quad (34)$$

$$j_w = \varrho T \Pi, \quad \Pi = \frac{w}{T} u_c F_L \left[ -\sqrt{3/2} \alpha_S \Lambda \frac{T}{w} \frac{\partial s}{\partial r} \right], \quad (35)$$

$$\frac{1}{\Lambda} = \frac{1}{\alpha_{\text{ML}} H_p^{\text{stat}}} + \frac{1}{\beta_r r}, \quad H_p^{\text{stat}} = \frac{p}{\varrho} \frac{r^2}{GM_r}, \quad (36)$$

$$\tau_{\text{rad}} = \frac{c_p \kappa \rho^2 \Lambda^2}{4\sigma T^3 \gamma_R^2}, \quad D_{\text{rad}} = \frac{\omega}{\tau_{\text{rad}}} \quad . \quad (37)$$

In the time-independent and static limit this is essentially mixing-length theory and the accuracy is assured by fitting the prescription to the Sun via a solar model. The difference is that the parametrisation is now brought into a fluid-dynamical framework and basic physical plausibility constraints that are required in the time-dependent regime are fulfilled (cf. Wuchterl and Feuchtinger (1998)). The Schwarzschild-Ledoux criterion is contained in the formulation via  $-\partial s/\partial r = c_p/H_p(\nabla - \nabla_s)$  and  $\nabla_s = \nabla_{\text{ad}}$  in the absence of energy sources and sinks inside eddies. Convectively unstable stratifications occur in this model when pressure and temperature gradients have the same sign and produce a positive value of  $S_\omega$  that then contributes a source of turbulent kinetic energy,  $\omega = 3/2 u_c^2$  to the balance equation of turbulent kinetic energy, Equ. 32.  $u_c$  being the convective velocity corresponding to mixing length theory. A general problem of mixing length theory — the violation of a convective flux-limit — has been corrected as described by Wuchterl and Feuchtinger by introducing a flux-limiting function (cf. Wuchterl and Tscharnuter (2003)). The great advantage of that approach is that a general prescription can be used for the Sun, stellar evolution, pulsating stars, brown dwarfs, planets and protoplanets. Any calibration of parts of the convection model obtained in one astrophysical system — the mixing length parameter calibrated by the Sun, the time-dependent behavior tested by RR-Lyrae stars — will decrease the uncertainties in applications to not-so-easy-to-observe systems as protoplanets.

## 17 Fluid-Dynamics of Protoplanets

The time-dependent convection model allows the formulation of a fully time-dependent set of equations (cf. Eqns. 15–20 discussed earlier in Sect. 6.4). These equations describe the radiative and convective envelopes of protoplanets as well as the protostellar collapse and pre-main sequence evolution (cf. Wuchterl and Tscharnuter (2003)).

The equations are applied to the volume taken by the protoplanetary envelope, and assuming spherical symmetry. They determine the motion of gas in the protoplanetary envelope or determine the hydrostatic equilibrium if forces

balance out. As a consequence of the structure and motion in the envelope the mass exchange with the nebula results and determines whether the planet gains or loses mass. Material belongs to the planet when it is inside the planet's gravitational sphere of influence. The sphere of influence is approximated by a spherical volume of radius  $R_{\text{Hill}}$  around the condensible element planetary embryo at the centre. At the outer boundary of the volume, i.e. at the Hill-sphere the protoplanet radiates into the ambient nebula and may exchange mass with it. Planetesimals enter the sphere of influence and collide with the core. The core's surface is the inner boundary. The surface changes its radius as the core grows due to planetesimal accretion. The planetesimals add mass to the core and dissipate their kinetic energy at the core surface. The core radiates the planetesimal's energy into the adjacent planetary envelope gas. That heats the inner parts of the protoplanetary envelope. The resulting temperature increase relative to the nebula induces temperature gradients that drive energy transport through the envelope towards the nebula. In general transfer occurs by both, radiation and convection.

## 18 Dynamic Diversity

Quasi-hydrostatic models of giant planet formation always find envelope-growth once the critical mass has been reached. When departures from hydrostatic equilibrium are allowed and the dynamics of the envelopes are calculated the situation is more diverse: the occurrence of accretion depends on the nebula properties and the properties of the protoplanet at the critical mass, see Wuchterl et al. (2000). Furthermore the onset of planetary envelope mass-loss depends on the planetesimal accretion-rate of the core and the treatment of energy transfer. The dependence is quantitatively significant on the scale of a few Earth-masses that is comparable to the masses of terrestrial planets, the cores of giant planets and the envelopes of planets like Uranus and Neptune. In Fig. 3 two calculations are compared for the Neptune position in the Kyoto-nebula<sup>15</sup>. Hence nebula-properties and orbital-radius-effects (orbital dynamic time-scale, solar tides, size of the Hill-sphere) are identical for both calculations. The difference is in the energy input and content of the envelopes, i.e. the thermal structure. The first calculation (full line in Fig. 3) is for a constant mass accretion-rate and simple instantaneous zero entropy gradient convection. The second calculation (dashed line) is physically more refined, with a particle-in-box planetesimal accretion-rate and time-dependent convection as described above. The gravitational focusing factor is chosen appropriate for the runaway phase up to the isolation mass. The outcome is qualitatively very similar to the one for simpler physics: with growing core mass the envelope mass increases until the slope becomes almost vertical in the vicinity of the critical mass (Fig. 3, left panel). But

<sup>15</sup>Mizuno's Neptune (17.2 AU, 45 K,  $3.0 \cdot 10^{-13} \text{ g/cm}^3$ ), for orbital, radius, nebula temperature and midplane nebula density, resp., is located inside of Neptune's present orbital radius (semi-major axis 30.06 AU) to allow for outward migration after formation (cf. Hayashi et al. 1985).

the values of the critical core mass and the envelope mass at given core mass are significantly different (critical core masses, 13 and 4  $M_{\text{Earth}}$  with envelopes of 5 and 0.5  $M_{\text{Earth}}$  respectively). The evolution beyond the critical core mass is shown as a function of time in the right panel. Note that the evolution is now on the short dynamical time-scale (a few years) of the envelopes. Mass-loss is driven in both cases, and both calculations approach a new quasi-equilibrium state with smaller envelope mass. But the envelope masses ultimately differ by approximately a factor of ten. Even with the relatively well known properties of giant planets at the critical mass, no general conclusion is possible about the dynamical processes that happen thereafter and the expected envelope mass of e.g. a Uranus-type planet. It is obvious that a more general understanding is needed to predict the outcome of planet formation when realistic physics, as runaway planetesimal accretion, dynamical effects and plausible convection are included.

Following the usual approach for solar system planet formation we might try adjust the parameters of planetesimal accretion to account for the observed properties of Neptune, say, but that will not lead to a predictive theory or a general understanding of planet formation. I will outline an alternative approach below.

## 19 A Few Problems of Solar System Theory

To conclude the discussion of solar system planet formation theory I will describe open problems that were known before the discovery of the first extrasolar planet. These problems might help to understand what parts of the theory might need modifications for the general application to planet formation in the galactic neighborhood. With dust growth to cm size now increasingly well understood by theoretical and experimental work, Blum and Wurm (2000) the most important remaining problems are:

1. planetesimal formation,
2. the total growth times in the outermost solar system, and
3. the final planetary eccentricities.

### 19.1 Planetesimal Formation

Planetesimal formation by coagulation and agglomeration of dust grains may stall at dm to m size where loss processes by radial drift may halt the planet formation process. Planetesimal formation by a gravitational instability of a dust-subdisk may require special nebula conditions that are incompletely explored to decide under how wide a range the instability will operate and whether the non-linear outcome are the consolidated condensable element bodies that are envisaged and assumed in the planetesimal hypothesis. The related key question is how wide a diversity of nebulae will lead to instabilities that produce appropriate planetesimals. Being appropriate mostly means a size large enough to

decouple from the head-wind of the nebula gas. An event that typically occurs at km size. Production of non-standard planetesimals does not automatically mean that planet formation will not proceed as presently imagined but new pathways in a theoretically essentially unexplored regime have to be worked out in that case.

## 19.2 Late Accretion: Total Planetary Growth-Times

The standard model is centered around the planetesimal hypothesis that has been successful to understand a wide range of solar system bodies, to a large extent in a quantitative way. But observational results obtained for nearby star-forming and young star regions quantitatively challenge the standard model because indicators of the presence of circumstellar disks Haisch et al. (2001) suggest disk depletion time-scales that are comparable or shorter than calculated formation times for solar system giant planets of at  $\approx 10^8$  years (Safronov (1969)). Moreover, unless the eccentricities of the growing embryos are damped substantially, embryos will eject one another from the star's orbit (Levison et al. (1998)). Runaway growth, possibly aided by migration (Tanaka and Ida (1999)), appears to be the way by which solid planets can become sufficiently massive to accumulate substantial amounts of gas while the gaseous component of the protoplanetary disk is still present (Lissauer (1987), Kokubo and Ida (2002)). The theoretical estimates for planetary growth times have been known to be idealized because the size distribution of planets, embryos and planetesimals and the interaction with the residual nebula gas can only be incompletely accounted for in then n-body calculations that are necessary to reliably calculate the final orbital outcome at least for an idealized situation to allow a quantitative discussion and theoretical progress. Inaba and Ikoma (2003) and Inaba et al. (2003) looked at the collisional cross sections of planetesimals with gaseous envelopes and found a significant increase for their accretion-rates reducing the planet-growth times considerably. This is especially important for the giant planet regime where the envelopes may become comparable in mass to the condensible element cores during the runaway phase. Hence total growth times can be expected to decrease further when the nebula gas is not neglected in determining the collision cross sections of planetary embryos.

## 19.3 Late Accretion and Final Eccentricities

Late accretion and hence the evolution to the final orbital parameters of a planet is governed by interactions with other planets, the remaining planetary embryos and planetesimals (Levison et al. (1998), Thommes and Lissauer (2003), Levison and Agnor (2003)). The relevant overall masses in all components may or may not be comparable to the mass of the largest planet. There is probably still a large and locally dominant number of bodies around, that in case of the solar system are responsible, e.g. for the formation of the moon and the late heavy bombardment. Late accretion effects are apparently important in the asteroidal region of the solar system where it is possible that Jupiter's perturbations

precluded the accretion of embryos into a planet. The important remaining dynamical process is then the orbital evolution of planets and embryos due to secular mutual perturbations. In models of final planetary growth they typically lead to eccentricities larger than observed in the solar system (Wetherill (1990), Chambers and Wetherill (1998)) and found in long-range backward integrations of the planetary system, Lecar et al. (2001). Studies with an increased number of planetary embryos reduce the discrepancy (Chambers (2001)) but still do not reproduce the low time-averages of the planetary eccentricities in the solar system. Eccentricity damping by residual nebula gas or by a remnant population of planetesimals or small planetary embryos might resolve that problem.

## 20 Theory-PlanetMania

In addition to the solar system problems, the planetary properties of the first exoplanet harvest were unpredicted by theory and surprising because of the detection of:

1. giant planets with orbital periods of a few days, corresponding to 0.01 of Jupiter's orbit-radius,
2. planet candidates with  $M \sin i$  up to  $13 M_{\text{Jupiter}}$ <sup>16</sup>,
3. a broad range of eccentricities larger than known for the solar system planets<sup>17</sup>,
4. planets in binaries<sup>18</sup>.

The 1995 discovery of a planetary companion to 51 Peg electrified theorists. Very rapidly Guillot et al. (1996) showed that planets like 51 Peg b could indeed survive for the estimated ages of their host stars. Within a year it was shown that 51 Peg b could form at its present location when existing fluid-dynamical models of giant planet formation were applied to orbital distances of 0.05 AU, provided sufficient building material was within the planet's feeding zone, Wuchterl (1996).

But very rapidly alternative theories emerged. They kept the often communicated view that giant planets would only be able to form beyond the ice-line, typically beyond a few AU from their parent star. If that remained true the planets had to move from their formation place to a position much closer to the star, like 51 Peg b's. How could a massive planet like Jupiter move from 5 to 0.05 AU, say? Mechanisms to change the orbital elements were proposed:

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<sup>16</sup> $1 M_{\text{Jupiter}} = 317, 71 M_{\text{Earth}} = 1.898 \cdot 10^{27} \text{ kg} = 0.95 \cdot 10^{-3} M_{\odot}$

<sup>17</sup>That might have been expected because of the difficulties to explain the low eccentricities but was not predicted. Most likely because missing elements in late accretion were obvious (see above).

<sup>18</sup>In spite of the fact that dynamicists had shown planetary orbits to be stable in binary systems as well as already classified them as P- and S-type (Dvorak (1986), Dvorak et al. (1989) in analogy to planets and satellites in the solar system

1. violent dynamical relaxation of multi giant planet systems the so called *jumping Jupiters*, and
2. a gradual decrease of the planetary orbital radius due to interaction with the disk of gas and planetesimals: *orbital migration*.

Violent dynamical relaxation (Weidenschilling and Marzari (1996); Marzari et al. (2005)) needs synchronising of planet formation to provide a number of giant planets within a narrow time-span. They would subsequently very rapidly interact via mutual perturbations that typically destroy the system on a dynamical time-scale leaving a close-in giant planet in some of the cases. While it is unlikely that the assumed very unstable initial state would be reached as the final state of the preceding planet formation process, there are additional problems. The close orbits produced typically would not be as small as observed, hence requiring further orbital evolution and the final systems would be rather disturbed with one close in and one planet far out. That is unlike a system like *v Andromedae* with relatively close orbiting giant planets in addition to the *Pegasi-planet* at 0.05 AU.

The other alternative, involving migration caused by disk-planet-interactions, is favored by many researchers (e.g., Lin et al. (1996); Trilling et al. (1998)). It starts out with a standard situation of planet formation: a planetary embryo or proto-giant planet orbiting at a conventional giant planet orbital distance. The change of orbital radius is continuous to very small values and there is now requirement of other planets to be present at the same time. I will discuss migration as the dominating theory of *Pegasi-planet* formation below.

Even more radical rethinking of planet formation has been proposed. It was reconsidered that giant planets might form directly via a disk instability (see Wuchterl et al. (2000) for a review). While this was more directed towards the time-scale problem of planet formation in the outer solar system it also might offer a way to explain the diversity in the detected extrasolar planets. Maybe some of the systems, in particular the very massive planetary candidates, with minimum masses  $M \sin i \sim 10 M_{\text{Jupiter}}$ , were formed by a disk instability and others by the nucleated instability?

With the formation process reconsidered, the relatively large minimum masses of many of the early exoplanet discoveries, the large eccentricities, that are hardly distinguishable from those of binary stars, and the *Pegasi-planets* the old question was re-posed: What is a planet?

## 21 What is a Planet?

Given the unexpected properties of extrasolar planet candidates and claims of discoveries of so called *free floating planets* the IAU's working group on extrasolar planets<sup>19</sup> issued a preliminary working definition<sup>20</sup> based on the following principles:

<sup>19</sup><http://www.dtm.ciw.edu/boss/IAU/div3/wgesp/>

<sup>20</sup><http://www.dtm.ciw.edu/boss/IAU/div3/wgesp/definition.html>

- objects with true masses below the limiting mass for thermonuclear fusion of Deuterium that orbit stars or stellar remnants are planets,
- substellar objects above that mass are brown dwarfs,
- free floating objects in young star clusters with masses below the Deuterium limit are no planets.

With these guidelines for a definition, most of the discovered extrasolar planets are candidates because their true masses are not yet sufficiently well known. Due to the unknown orbital inclination only the minimum masses  $M \sin i$  are known. These values also probably approximate the true masses well, but a few % may turn out not to be below the  $13 M_{\text{Jupiter}}$  mass limit. Only the masses of the pulsar planets (from mutual perturbations) and the transiting Pegasi-planets, HD 209 458 b and TrES 1 (orbital inclination determined from transit light curve) are true masses in the sense of the above definition.

The IAU working definition also repeatedly and explicitly excludes the way of formation from the definition. But by referring to the Deuterium-limit the formation history implicitly enters the definition through a back-door.

The minimum mass for thermonuclear fusion of Deuterium is a concept that is shaped in some analogy to the minimum mass for hydrogen burning that defines the lower end of the stellar main sequence. But the main sequence is defined by the stellar thermal equilibrium, where nuclear burning fully balances the surface energy-losses. The stellar luminosity is balanced by nuclear energy production of the same magnitude. Objects with masses below the lower end of the main sequence also burn hydrogen, but insufficiently to fully balance the luminosity (Kumar (1963)) — they are called brown dwarfs. Because of that fact stellar thermal equilibrium and a phase of constant radius is never reached. The objects have to contract forever to (at least partly) supply their luminosity need from contracting into their own gravitational field. Because they never reach an equilibrium state (as the main sequence) their evolution always depends on their history. Ultimately that means it depends on their formation. Recent calculations of spherical collapse of stars and brown dwarfs show that the young objects after the end of significant mass accretion (on the pre-main sequence for young stars) contain the thermal profile shaped by the collapse, Wuchterl and Tscharnuter (2003). But it is exactly that thermal profile that controls the ignition of thermonuclear fusion processes. That is particularly true for Deuterium that is in all models (hydrostatic and dynamic) a very episodic event in the first few million years for stars *and* brown dwarfs. That means that for masses below the main sequence, the question of whether Deuterium (or hydrogen) will just start to burn — and that defines the borderline in the IAU-definition — will depend on the history of the respective low mass object. For Deuterium, that burns early on, that means it will depend on the formation process.

In summary, *hydrogen burning* to define the lower end of the main sequence means using a major, physically dominating process that defines a long-lasting equilibrium state that contains no memory about the formation history. To

the contrary using *deuterium burning* for characterizing a planet means using an essentially irrelevant process for the evolution of low mass objects, that is history-dependent in a way that will be very hard to predict.

I think we should rethink the definition of a planet along the following mayor characteristics:

1. mass,
2. heavy element enrichment,
3. orbital stability properties,

The first item is straightforward with the large enrichments (bulk and atmospheric) of planets relative to their parent star. A factor of 3 and more should also be a working basis, that is empirically very likely much less challenging in terms of future determinations in exoplanets than trying to observe the presence of Deuterium in a few Jupiter-mass companion in a 10 AU orbit, even around a nearby star.

The second point is still hard to characterize quantitatively but large progress has been made in the understanding of the stability-properties of the solar system, Lecar et al. (2001). The basis could be volume exclusion principles based on the non-overlap of planet-domains with a width of multiples of the Hill-radii. Laskar (2000) has recently shown that they are the consequence of simple assumptions about planetary growth via pairwise embryos collisions. The low solar system planetary eccentricities could be a special case of that. Certainly low eccentricity planets can orbit closer together in terms of Hill-exclusion.

The third point is the most uncertain. Observationally the characteristic mass of the detected planetary population seems to decrease as more discoveries are made. Presently it may be around 3 Jupiter-masses, with the estimated true distribution still peaked towards the detection limit. Theoretically the planetary masses are presently essentially unconstrained at the upper end. The best hypothesis for the moment is that planetary masses are limited by the amount of material that is within the respective feeding zone in the nebula. This definition contains a considerable degree of circularity in general but has been consistently worked out at least for planetary embryos.

In summary, I think we will see the definition of a planet remaining a *morphological type*, i.e. without an explicit, constructive definition for some time. But I think that condensible elements should play the major role, not a hydrogen trace-isotope.

## 22 Why Not Abandon Solar System Formation Theory?

If the extrasolar planet properties are so different, and the theories developed for the solar system did not predict their properties, why not look for a completely new theory of extrasolar planet formation? Should this new theory be more

along the lines of binary star formation? As one might conclude given the fact that the period-eccentricity distribution of extrasolar planets is indistinguishable from the one of binary stars.

I think we should not throw away the solar system formation theory as a general theory of planet formation too early. It does not only provide a fairly consistent picture of solar system bodies ranging in size from interplanetary dust particles to Jupiter, but it has also led to the development of predictive elements that led e.g. to the correct prediction of many orbital properties of trans-neptunian objects.

On the other hand *szenaria* like the *jumping Jupiters* to explain Pegasi planets raise more questions than they answer. Instead of one planet at 0.05 AU the simultaneous formation of many massive planets is required as a presumption.

Moving a planet in place by migration, requires in addition to the planet a mechanism that counteracts the migration process to *park* the planet once it has arrived at the intended final orbit. That is by no means trivial because of the large migration-rates (on the local disk evolution time-scale and shorter) that increase as the star is approached resulting in acceleration rather than slow-down for small orbital radii. Numerous *parking*-processes have been suggested, but ultimately the way out of the dilemma might be only the dissipation of the nebula. Planets would then continue to form, drift inward, and disappear into the star until the exhaustion of nebular material finally ends this road of destruction. Observationally no metal trend versus effective temperature is found on the main-sequence, reflecting the different sizes of convection zones that would play the role as a planetary graveyard and hence might be expected to be heavy-element enhanced, Santos et al. (2003).

The disk instability model, if it works and does indeed form planets, as a general alternative would have to be augmented by a separate way of terrestrial planet formation. For the giant planets the disk instability would probably require a separate heavy element enrichment process. Even if Jupiter formed by a disk instability, the craters on a Galilean moon would recall the planetesimal picture.

The basic feature of the non-standard planet formation theories is that they quickly provided scenarios for newly discovered objects. But typically they would fail the solar system test. Let us look at planetary migration as an example of the new pathways of planet formation, and discuss it in more detail.

## 23 Planet-Disk Interaction

### 23.1 What is Planetary Migration ?

Planetary migration presently seems to denote any systematic change of the orbital semi-major axis of a planet that does not change direction. Historically outward migration of Uranus and Neptune as a consequence of ‘passing comets down to Jupiter’ seems to be the first large scale post-formation reshaping process of planetary orbits that was considered. It had been noticed by Fernandez

and Ip (1984) and has been considered by Hayashi et al. (1985) as a process that would allow shorter growth times for Uranus and Neptune. In the late stages of outer solar system formation these planets would move outward as a consequence of angular momentum exchange when perturbing comets into a Jupiter controlled orbit with subsequent ejection to the Oort cloud.

After the discovery of 51 Peg b it has become a custom in planet formation theory to denote many kinds of changes in the planetary semi-major axis or orbital distance as planetary migration. That is usually independent of the physical process underlying the respective orbital change. With processes proposed and a terminology of types I and II, suggested by Ward (1997), and a type III added later by analogy we have in particular:

**type I migration:** an embedded planetesimal or planetary embryo that interacts with its own disk-density-waves;

**type II migration:** a protoplanet that has opened a gap in the nebula — i.e. produced a region of reduced nebula density in its feeding-zone — is locked in that gap and follows the gradual inward motion of viscous disk gas together with the gap;

**type III migration:** an instability of the planet-disk-interaction that leads to orbital decay within a few orbital periods.

We distinguish here between migration processes that modify the orbit by less than a factor of  $e^2$  (or  $\sim 10$ ) and those that may lead to larger changes up to orders of magnitude in the orbital radius, and may ultimately result in the loss of the planet. The latter processes we will call *violent migration* in the following. They may dominate the planet formation processes if they operate in many and diverse nebulae.

After planetesimal formation violent migration is the second key problem of planet formation. Like an inefficient planetesimal formation mechanism it has the potential to make the formation of systems similar to the solar one, very unlikely. It is expected by many investigators to become important in the mass range resulting from the early fast *runaway*-mode of planetesimal growth. The runaway-phase ends when all planetesimals that are within the gravitational range of the locally largest body have been accreted and hence its feeding zone has been emptied. Planetary embryos gravitationally interact with the ambient gas disk, planetesimal disk and other planetary embryos or planets. As a result *planetary migration* can come about (see Thommes and Lissauer (2005), for a review).

The migration effects become severe at larger sizes because they are proportional to the planetesimal mass, for type I (after Thommes and Lissauer (2005), cf. Ward (1997)):

$$v_I = k_1 \frac{M}{M_*} \frac{\Sigma_d r^2}{M_*} \left( \frac{r\Omega}{c_T} \right)^3 r\Omega \quad (38)$$

where  $k_1$  is a measure of the torque asymmetry,  $M$ ,  $M_*$  the masses of the planet and the primary resp.,  $r$  is the orbital radius,  $\Omega$  is the disk angular velocity,

that is approximately keplerian with  $\Omega_{\text{Kepler}} = \sqrt{GM_*/r^3}$ ,  $\Sigma_d$  the disk surface density and  $c_T$  the isothermal sound speed. For a planet that has opened a gap and is locked to the disk the rate of change in orbital radius (type II migration) is Ward (1997):

$$v_{II} = k_2 \frac{\nu}{r} = k_3 \alpha \left( \frac{c_T}{r\Omega} \right)^2 r\Omega, \quad (39)$$

where a nebula viscosity  $\nu \sim \alpha c_T^2 / \Omega$  has been assumed, and  $k_2$  and  $k_3$  are further constants. For easier reading it is worth noting that the vertical disk-scale height  $h \sim c_T / \Omega$  and  $h/r$  is roughly constant and  $\sim 0.1$  in some nebulae. Note that both rates are proportional to the Keplerian orbital velocity  $r\Omega$ .

## 23.2 Violent Migration

Violent migration is a back-reaction of the planetary embryos ‘bow wave’ in the nebula onto the embryo itself. As the embryo orbits the star, its gravitational potential adds a bump to the stellar one. At the embryo’s orbit — at the corotation resonance, in the linear terminology of migration theory — the embryo and its potential move almost at the same, keplerian, velocity. That is co-orbital motion as in the case of Jupiter and the Trojan asteroids. Inside the embryo’s orbit, the gas in a quasi-keplerian disk orbits faster and hence the embryos potential and gravitational acceleration travels at a different speed relative to the gas. This accelerates the disk gas and excites a pressure- and density-wave that travels with the embryo. Because matter deeper in the primaries potential must orbit faster the waves are dragged forward inside, and backwards, relative to the embryo, outside the embryos orbit. These rather particular protoplanetary bow-waves include density enhancements that gravitationally back-react onto the planet. Due to the inherently asymmetric nature of the situation (keplerian orbital velocities changing  $\propto r^{-1/2}$ ) and the particular wave-pattern the forces (and in particular the torques) on the planet may not cancel out. They do not cancel out for simple disk models like plausible radial power-laws for the nebula surface density. That leads to a net exchange of angular momentum between the planetary embryo and the gaseous disk if the waves dissipate or break in the disk, neighboring the planet’s gravitational sphere of influence. The result is the familiar reaction of orbiting matter if angular momentum exchange is allowed: most matter (the embryo) moves in and a small amount (some gas) moves out carrying away the angular momentum. The very growing of the embryo would lead to orbital decay and gradual movement towards the star on time scales of disk evolution or much smaller. Many studies are presently devoted to determine the strength of the effect and evaluate the rates of orbital decay and hence the possible survival times for planets of given mass in a given disk. If migration dominates planet formation, it has the potential to wipe out any and many generations of planets. In that case and because the basic effect originates from a relatively small difference in a delicate torque balance, in a significantly perturbed non-keplerian disk, I doubt that we will be able to reliably predict much about planet formation any time soon.

### 23.3 A Closer Look

Modern planetary migration theory originated from the study of planetary rings (cf. Ward (1997)). While the basic physical processes, density waves in quasi-keplerian disks are well studied, the application to the problem of forming planets in the nebula disk is not straightforward.

The basic problem that has to be solved to determine migration-rates for a proto giant planet orbiting in a nebula disk is the fluid dynamical analogue to the restricted three body problem of celestial mechanics. In the classical problem of celestial mechanics the motion of a test particle is considered in the combined gravitational field of the Sun and a planet. For a proto giant planet two modifications have to be made:

1. a protoplanet, unlike a mature planet is not well approximated by a point mass,
2. the test particles are replaced by a fluid with a finite pressure.

A protoplanet fills its Hill-sphere and a considerable fraction of its mass is located at significant fractions of the Hill-sphere (e.g. Mizuno (1980), Wuchterl and Pečnik (2005)). Furthermore the protoplanet builds up a significant contribution to the gas pressure at the Hill-sphere. Typically planet and nebula are in a mechanical equilibrium. This may only change when and if the planet collapses into the Hill-sphere and does not rebound. Fluid dynamical calculations show that this is a non-trivial question that depends on the structure of the outer protoplanetary layers, near the Hill-sphere Wuchterl (1995a). In consequence, the problem of a protoplanet in a nebula disk is not only a problem of gas-motion in the gravitational potential of two centres, but it is controlled by the nebula gas-flow and the largely hydrostatic equilibrium of the protoplanets themselves. The Hill-spheres are filled by hydrostatic protoplanets at least up to the critical mass,  $20 M_{\text{Earth}}$ , say, and by quasi-hydrostatic structures, typically until  $50\text{--}100 M_{\text{Earth}}$ . In fact strictly static solutions for protoplanets are published up to masses that closely approach that of Saturn, Wuchterl (1993). Static isothermal protoplanets may be found with masses comparable to Jupiter's, Wuchterl and Pečnik (2005). As a consequence the protoplanetary migration problem is very far from the idealisations of essentially free gas motion in the potential of two point masses.

Because the problem is basically three-dimensional and the density structure of a protoplanet covers many orders of magnitude, additional approximations have to be made to solve the problem — either numerically or analytically. The basic analytical results (see Ward (1997)) stem from solving the linearised fluid dynamical equations for power-law nebula-surface densities and an approximate gravitational potential of the problem. The starting point is an unperturbed, quasi-keplerian disk. The planet is approximated by an expansion of the perturbations induced by point mass. The linear effect (spiral density waves launched in the disk) is deduced and the resulting torques of the waves on the planet are calculated, assuming how the waves dissipate (break in the disk). If the waves

(and the angular momentum carried) would be reflected and return there would be no effect. This approach has at least two potential problems:

1. the dense parts of the protoplanets, in the inner half of the Hill-sphere, say, that potentially carry a large fraction of momentum are treated as if there were no protoplanet — the density structure of the disk is assumed to be unperturbed by the protoplanetary structure even at the position of the planet’s core, certainly throughout the Hill-sphere. In that way the pressure inside the Hill-sphere is dramatically underestimated. The Hill-sphere effectively behaves like a hole in the idealized studies of the problem: the gravity of the protoplanet is introduced into the calculations, but the counteracting gas pressure of the static envelopes is omitted.
2. the unperturbed state, that is needed for the linear analysis is an unperturbed keplerian disk. But if a planet with finite mass is present, the unperturbed state is certainly not an axially symmetric disk and corrections have to be made at all azimuthal angles along the planets orbit. The keplerian disk in the presence of a protoplanet or an embryo is an artificial state that is found to decay in any non-linear calculation. Certainly it is not a steady state as would be required for a rigorous linear analysis. Therefore the approach is not mathematically correct. It may turn out that the corrections are minor, as in the case of the *Jeans-swindle*, and the basic results hold despite considerable mathematical violence. But unlike in the Jeans case, where Bonnor-Ebert-spheres show that there are indeed nearby static solutions, nothing similar is available for the planet-in-disk problem. In fact the respective steady flows are essentially unknown and it is questionable whether they exist at all in the fluid dynamical problem — their might always be a non-steady planetary wake trailing the planet. High resolution calculations, Koller and Li (2003); Koller et al. (2003) indeed show considerably vorticity and important effects on the torques at the corotation resonances with potentially important consequences for the migration-rates.

Nonlinear 3D (and 2D) hydrodynamic calculations of the problem are very challenging, both in terms of time-scales and spatial scales. The ‘atmospheric’ structure of the protoplanet inside the Hill-sphere can barely be resolved even in the highest resolution calculations (D’Angelo et al. (2002, 2003, 2005)) and the dynamics has to be done for simplified assumptions about the thermal structure and dynamical response of the nebula (usually locally isothermal or locally isentropic). The great value of this calculations is that they provide information about the complicated interaction of the planet with a nebula disk that can only be incompletely addressed by models with spherical symmetry that calculate the structure and energy-budget of the protoplanet with great detail. The interaction regime between the outer protoplanetary envelope, inside half a Hill-radius and the unperturbed nebula disk, at five Hill-radii, say, is only accessible by 2D or 3D calculations. It is unknown due to the lack of any reference flows, be it experimental or theoretical. This situation in my opinion is similar to the one

of the restricted three body problem, at the time when numerical integrations just started.

Hence migration-rates calculated numerically or analytically have to be considered preliminary and await confirmation by more complete studies of the problem. Agreement that has been found between different investigations is within the very considerable assumptions outlined above and does not preclude considerable uncertainties in the migration-rates by many orders of magnitude.

### 23.4 Is a Planet a Hole or Not a Hole?

To illustrate the progress, that has been made by high-effort, state-of-the-art high resolution calculations, I briefly want to discuss a set of new calculations (D'Angelo et al. (2002, 2003)) that have brought considerable insight into the problem of how accretion into the Hill-sphere of a protoplanet may occur if the protoplanet is assumed to accrete the gas into a small area, essentially onto a mature planet D'Angelo et al. (2003). For the first time the planet was not assumed to be a point mass that accretes everything that approaches the limiting resolution of the calculation but alternative assumptions were made about central smoothing of the protoplanetary gravitational potential guided by analytical structure models (Stevenson (1982) and Wuchterl (1993)) in order to look at the pressure feedback of the growing protoplanet onto the accretion flow. The flow in the Hill-sphere turned out to be qualitatively and quantitatively very different for the two assumptions: with and without the pressure build up by the protoplanetary envelope — or in short with, or without a hole. The results demonstrate that the planetary structure feeds back on the flow, inside and also *outside* the Hill-sphere. Migration-rates derived from the 3D calculations were, depending on planetary mass, reduced down to 1/30 of the respective Ward (1997) analytical values.

It is important to add a note of caution to the interpretation of the planetary masses or mass-scaling used in the 2D and 3D calculations of planet-disk-interaction and planetary migration. The scale of the critical mass for isothermal protoplanets with typical nebula temperatures (100 K) is  $\sim 0.1 M_{\text{Earth}}$ , i.e. about a factor 100 below the 'realistic' values of  $7 - 10 M_{\text{Earth}}$ , that are typically found in detailed planetary structure calculations. Hence the typical regimes calculated in higher dimensional isothermal studies are a factor 100 supercritical! An isothermal dynamical calculation of planet-disk interaction, accretion and migration, for  $10 M_{\text{Earth}}$  hence roughly corresponds to the accretion of a protoplanet of 3 Jupiter-masses, a Jupiter-mass isothermal case to a realistic one with  $0.1 M_{\odot}$ ! The effective mass-scale of isothermal 2D/3D studies is *10 to 3000 times* supercritical. That scaling relates to all parts of the calculations within the gravitational range of the planet, a few Hill-radii, with the most severe effects located inside the Hill-sphere, in the protoplanetary envelope. Hence published studies of disk-planet interactions are presently in a much more violent regime than detailed 1D planet growth models require! An overlap is technically challenging but needed: studies at same effective physical scale that is set by the critical mass for the appropriate thermal planetary structure.

Calculations that treat the protoplanet *and* the planet nebula interaction in detail, i.e. by accounting for the heating- and cooling-processes as well as realistic thermodynamics at the required resolution and over a significant fraction of the planetary growth time are still in the future. But a first study of the coupled problem seems to be within reach for the idealized isothermal case.

## 24 Towards a General Theory: In Search of the Planetary Main Sequence

The isothermal case is well studied in higher dimensional calculations of planet-nebula interaction but comparatively little attention has been paid to isothermal models of the structure of protoplanets (e.g. Sasaki (1989)). The reason for that is the isothermal assumption for protoplanets is physically unrealistic: the atmospheres of giant planetary embryos become optically thick very early in the growth (Mizuno (1980)) and even for extreme assumptions about the metallicity, Wuchterl et al. (2000). The advantage of isothermal models is that they are comparatively easy to understand.

Motivated by unsolved problems of detailed statical and dynamical models — the physical nature of the critical mass and unpredicted equilibrium structures found in dynamical models (Wuchterl (1991a,b, 1993); Wuchterl et al. (2000)) — Wuchterl and Pečnik (2005) classified all isothermal protoplanets to identify possible start- and end-states of dynamical calculations.

### 24.1 All Isothermal Protoplanets

The construction of all possible isothermal hydrostatic protoplanets is analogous to the construction of the main sequence for stars — both are defined by equilibria. Because of the nature of the equilibria they are independent of the history that leads to them. For the main sequence the stellar equilibria are long-lived and hence describe most of the observed stars. For planets, a similar survey to look for all equilibria has not been performed. Our knowledge presently is one of the end-states of planetary evolution — the compact cooled planets. Their best stellar analog may be white dwarfs. But in their youth, planets had rich and long lasting equilibria that have not been explored from a global point of view. First steps in the construction of a *planetary ‘main-sequence’* — the most probable planetary states in the nebula — have been started now. For the isothermal case, Wuchterl and Pečnik (2005) not only found the end-states (mature planets and the planetary embryo states) but a large number of previously unknown planetary equilibria. They found multiple solutions to exist in the same nebula and for the same protoplanetary embryo’s core mass. Those calculations can now be used to constrain isothermal 2D and 3D calculations such that consistent overall solutions of the planet formation problem may be found.

For the first time analogous to the stellar main sequence all protoplanets are known for the isothermal case and a statistical discussion in a diversity of

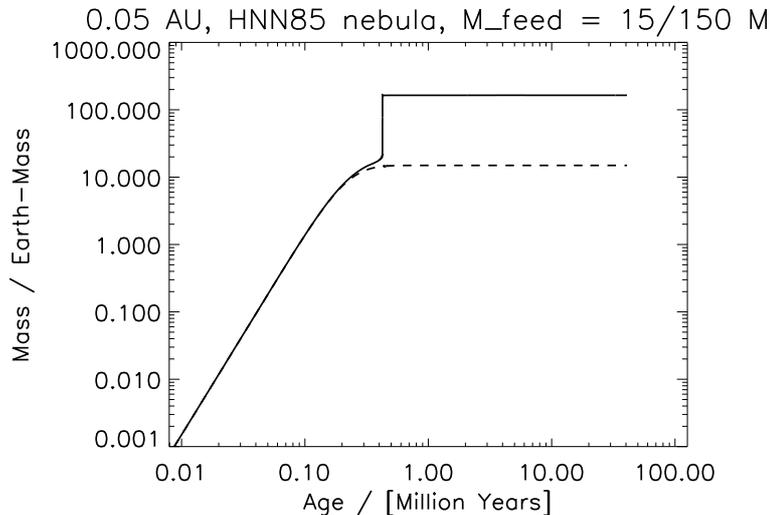


Figure 4: Total mass (full line) and core mass (dashed line) of a Pegasi-planet forming at 0.05 AU from a solar mass star. The accumulation of a gaseous envelope surrounding a condensible element core that grows by planetesimal accretion is shown for the first 100 Mio years. The structure of the envelope is calculated dynamically including time-dependent theory of convection that is calibrated to the Sun and including detailed equations of state and opacities.

nebula as well as a classification of the pathways of planet formation are now possible.

## 25 Formation of Pegasi-Planets

To finish the theoretical considerations I will give an example of a complete formation history of a planet from planetesimal size to its final mass. I will briefly discuss the formation and early evolution of a Pegasi-planet, from 0 to 100 Ma.

### 25.1 A Pegasi-Planet: Formation and Properties

The first extrasolar planet discovered in orbit around a main-sequence star was 51 Peg b, Mayor and Queloz (1995). With a minimum mass  $M \sin i = 0.46 M_{\text{Jupiter}} \sim 146 M_{\text{Earth}}$ , a semi-major axis of 0.0512 AU, a period of 4.23 days and an eccentricity of 0.013 it is the prototype of short period giant planets, the *Pegasi-planets*.

To model the formation of such an object I assume the midplane properties of a standard minimum reconstitutive mass nebula (Hayashi et al. (1985)) at

0.052 AU, i.e. 0.01 of Jupiter's semi-major axis and feeding zone masses of  $15 M_{\text{Earth}}$  of solids (sufficient to easily reach the critical mass) and  $150 M_{\text{Earth}}$  of nebula gas, respectively. This is motivated by the fact that the  $M \sin i$  value is only a lower limit for the mass and accretion may not be 100 % efficient.

With that assumptions, the equations of self-gravitating radiation fluid dynamics for the gas (Equ. 15 to Equ. 20) with time dependent convection (Equ. 21), calibrated at the Sun, are solved for a Hill-sphere that is embedded in the standard nebula at 0.05 AU. The condensible element core at the center of the sphere grows by planetesimal accretion according to a the particle-in-box rate. The minimum-mass solid-surface-density (a safe lower bound) and a gravitational focusing factor of 3 (Safronov-number of 1, also an assumption towards slow growth) are used.

## 25.2 Mass Accretion History – The First 100 Mio Years

The resulting mass accretion history from zero to 100 Ma is shown in Fig.4 with a logarithmic time-axis. Age zero is chosen, following Wuchterl and Tscharnuter (2003) at the moment when the envelope becomes optical thick for the first time and hence a thermal reservoir is formed. The calculation starts at a core of  $\sim 10$  km size and a mass of  $\sim 10^{15}$  kg. The displayed evolution in Fig. 4 starts 10 ka after the embryos envelope became optically thick, at roughly a tenth of a lunar mass. At that time the total mass (full-line) and the core mass (dashed) are essentially the same because the envelope mass is negligible. At 200 ka and somewhat below  $10 M_{\text{Earth}}$  the two curves separate, due to a gaseous envelope of significant mass developing. The planetesimal accretion-rate at that time has already dropped due to depletion of the solids, and the core-growth-curve  $M_{\text{core}}(t)$  starts to flatten out, with the total mass following. As the critical mass is approached, the total mass curve turns upward with the core-mass flattening further. That shows the onset of efficient envelope accretion. The contraction of the envelope is still quasi-static and the gas is practically at rest. The step in the total mass reflects a period of efficient envelope accretion that rapidly increases the total mass until the feeding zone is essentially emptied. The Mach-numbers are finite during this stage, but the hydrodynamical part of the flow is basically a transition flow from the nebula onto the contracting inner parts of the protoplanetary envelope that are quasi-hydrostatic. After the flow from the feeding zone onto the planet has faded, the masses remain constant - a Pegasi-planet is born.

## 25.3 Luminosity of a Young Pegasi-Planet

The luminosity of the Pegasi-planet corresponding to the mass-accretion history above, is shown in Fig. 5. The luminosity increases during mass-growth, passes through a double maximum and then decays roughly exponentially. The two luminosity maxima reflect the maximum accretion of solids and gas, respectively. Initially the growth, starting approximately at a  $nL_{\odot}$ , rises due to the planetary embryos increased, gravity enhanced cross section for planetesimal accretion. As

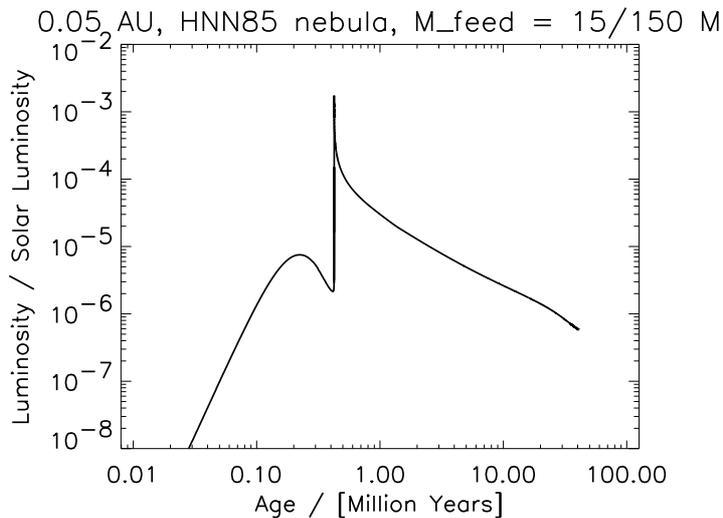


Figure 5: Luminosity of a Pegasi-planet during the first 100 Mio years.

the planetesimals are removed from the feeding zone and incorporated into the embryo, the surface density of the remaining condensible population fades and the luminosity turns over. Planetesimal accretion passes thermal control to the contracting gaseous envelope. As the envelope mass becomes comparable to the core its contraction controls the luminosity of the planet. With approaching the critical mass the luminosity turns upward again due to the rapid growth of the envelope reaching the sharp peak at maximum accretion. With the arrival at the final mass no further material is added and the only luminosity supply is contraction of the envelope, that slows down as larger parts of the planet degenerate. Thereafter the planet cools into the present with its luminosity being inverse proportional to age.

Most of the planetary evolution turns out to be quasi-hydrostatic with a brief dynamical period around maximum accretion: the step-like increase in Fig. 4 and the narrow luminosity peak in Fig. 5. During this brief period, most of the mass is brought into its final position and acquires its initial temperature. The rapid, dynamical phase is so fast, that there is essentially no thermal evolution occurring. Hence it sets the initial thermal state, and determines the bulk starting properties of the planet's evolution at its final mass.

The luminosity of the planet during this period lasting a few hundred years at an age of a few hundred thousand years is shown in Fig. 6. The entire evolution shown in this Fig. 6 is present in Fig 5 but unresolved in the luminosity spike. The peak accretion-phase starts at the turnover of the luminosity, cf. Fig. 6. Because the contraction is rapid the outer parts of the envelope are adiabatically cooled and the nebula gas starts radiating *into* the protoplanetary envelope. The

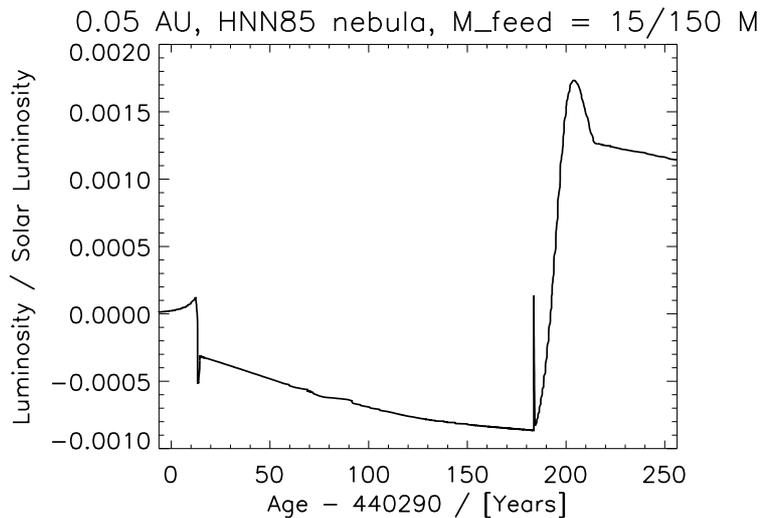


Figure 6: Luminosity of a Pegasi-planet: detail around maximum luminosity.

luminosity becomes increasingly negative as the inner, most massive parts of the protoplanet contract further. The trend is reversed when the contraction of the central parts are slowed down again and the heat produced in the process reaches the outer boundary of the planet for the first time (the brief spike at 180 years in Fig. 6). Then the overall contraction and accretion of the planet takes over again and the luminosity rises to positive values, reaching the peak at maximum gas accretion. As accretion fades the luminosity turns over and decreases with the vanishing amount of nebula gas remaining in the gas-feeding zone. At the end of significant gas accretion, the luminosity-slope sharply changes, as the luminosity becomes determined by contraction alone (beyond 220 a in Fig. 6). The initial luminosity of the planet at its final mass is  $\sim 1 \text{ mL}_{\odot}$  but fades rapidly. The total width of the spike in Fig. 5, at a tenth of the peak value, i.e. at  $\sim 10^{-4} L_{\odot}$  is appr. 100 000 a.

The final mass of the planet is approximately equal to the  $M \sin i$  of 51 Pegasi b. Being below a Jupiter mass (cf. Wuchterl et al. (2000)) there is a chance that spherical symmetry will give fairly correct results not only up to  $100 M_{\text{Earth}}$  but even for the gas accretion-rates towards peak-accretion and stagnation when the feeding zone is emptied. The rapid transition from 20 to  $150 M_{\text{Earth}}$  also may provide a way to escape violent migration by quickly removing any relevant, migration-driving mass from the nebula disk before significant orbital decay occurs in type-II migration mode.

The critical assumption concerning the above calculations is whether feeding zones with the assumed masses are plausible. In a minimum mass nebula, the mass integrated over plausible feeding zones is only a few earth masses. But

to understand extrasolar planets and planet formation in general the minimum mass nebula that is reconstructed from the solar system is a much too narrow constraint for the plausibility of the available mass. Most likely the nebula will be gravitationally stable during the planet formation epoch. That requirement allows to derive a more general constraint on the feeding zone. The gravitational stability of disks can be roughly estimated by the Toomre-criterion for axial symmetric gravitational stability. I suggest to use the marginally Toomre-stable nebula as the limit for the variety of nebula conditions that are allowed for consideration. The so obtained maximum midplane density values for planet forming nebula are a factor 200 above the minimum mass at 0.05 AU. In a very conservative feeding zone of one Hill-radius around the orbit, a Jupiter mass ( $318 M_{\text{Earth}}$ ) can easily be accounted for in such an enhanced nebula. Typical feeding zones would have more than five times the radius and hence provide a volume around the orbit that is 25 times larger. Hence many Toomre-stable nebulae can provide sufficient mass in plausible feeding zones at 0.05 AU to form a giant planet. The above calculations hence show, that giant planets form in less than a million years at 0.05 AU if their orbits are stable, i.e. migration-rates remain small.

Relying on a standard minimum mass nebula and planet formation fluid-dynamics that are physically improved but following a simple model-setup, it is possible to explain the formation of a Pegasi-planet in-situ, provided there is sufficient mass of gas and solids in the feeding zone. That is not the case for a minimum mass nebula. But assuming the diversity in extrasolar planets originates from a diversity in disk properties, we may vary the global nebula parameters. Gravitationally stable nebula that have less total angular momentum and hence more mass closer in, may provide sufficient mass to dynamically grow giant planets at 0.05 AU.

## 26 Is *it* Misleading or Not?

I have shown that one of the surprising discoveries, the Pegasi-planets can be reconciled with standard solar system formation theory, if a diversity of nebulae is accepted. I will now return to the empirical bases for the doubts raised about the general validity of solar system based understanding of planet formation: the exoplanet discoveries.

### 26.1 Is There No Bias ... ?

One of the first questions that immediately occurs is whether the discoveries are biased, as is very usual in astronomy because faint things are harder to see. Possible sources of bias introduced by the dominating radial velocity technique are:

1. sensitivity of the radial velocity measurements (highest for large masses and short periods),

2. the planet hunting grounds and hunting tactics,
3. the discovery race,
4. the binary issue, i.e. that the binary fraction in the galaxy is much larger than in the typical samples of radial velocity planet searches,
5. the selection of suitable host-stars to avoid variability, activity, youth, giants etc..
6. the extraction of reliable planetary signals from the data — two massive planets with widely separated orbits are easier to identify than two relatively low mass planets with comparable masses and relatively close orbits. That is especially true when the orbital periods are close to multiples of each other as in case of Jupiter and Saturn.

That questions can only be answered by very well-defined and complete samples and to some extent by other discovery methods. It is interesting to look at the planetary ‘yield’ of the ongoing transit-searches. They have by far not detected as many planets as would be (naively) expected from an extrapolation of the radial-velocity discoveries. It has to be seen whether this is due to difficulties of the transit method, or just due to different biases of radial-velocity and transit-searches. Clearly an overlap of methods is important and seems to be possible for astrometry and direct imaging within a few years.

## 26.2 Towards Normality

With the radial velocity method providing by far most of the information about extrasolar planets it is interesting to look at how the typical properties of the discovered exoplanets change as more and more are discovered. It seems notable that:

- the ‘outskirts’ of the eccentricity distribution approaches the solar system planets with an overlap in all parameters expected soon,
- the periods of the discovered planets increase with time, now starting to overlap with the solar system giant planets. Waiting seems to make the solar system more typical,
- the median of the distribution of minimum masses seems to continue to decrease. It seems that the characteristic mass has changed from about  $M \sin i = 7$  in 1996, to 4 in 2000 and I understand observer’s talks such that  $M \sin i = 2$  or 1 may be possible for the final outcome.

The next important step in the discoveries is an extrasolar planet that overlaps with a solar system giant planet in all properties, i.e. mass and eccentricity less than Jupiter’s, orbital period larger than Jupiter’s.

### 26.3 Brave Hearted Searches — Icarus!

To close the gap, searches are necessary where they are most difficult:

1. avoiding ‘hunting bias’, i.e. without a-priori input to select stars for planetary yield or assigning higher observation priority to ‘good’ stars, i.e. stars with low radial-velocity-‘noise’,
2. volume complete samples (see next section),
3. searches for planets in binaries (e.g.  $\alpha$ -Cen, Endl et al. (2001))
4. planets of stars with a type earlier than late F — most RV samples focus on stars later than that. But Setiawan et al. (2003) detected a planet around an K1III giant, a star that was an A-star on the main sequence, expanding the mass range of known stars with planets,
5. planets in clusters (as opposed to *cluster planets*), that have the advantage of a more homogeneous and coeval stellar populations,
6. searches for young planets to determine the earliest time at which planets exist,
7. the host star mass-range is also considerably widened by studying M-stars. Kürster et al. (2003) showed that planets with  $M \sin i$  of a few  $M_{\text{Earth}}$  could be detected around M-stars,
8. Guenther and Wuchterl (2003) went to the extreme and searched for planets around brown dwarfs. While significant RV-variation was detected only an upper limit could be set for the presence of Jupiter mass planets.
9. direct imaging searches to look for planets in long-period orbits and start of the direct characterization (Neuhäuser et al. (2000)).

### 26.4 Looking at Stars Near You — Metallicity

To show the present state of the discussion and possible problems with biases I will briefly discuss the planet-metallicity relation (Santos et al. (2003), Fisher and Marcy (2005)). The authors find the frequency of planets to increase with metallicity (i.e. the  $[\text{Fe}/\text{H}]$  metallicity indicator). Fuhrmann (2004, 2002) studied a volume complete sample of nearby F,G and K stars that overlaps with the planet hunting samples. When compared to the Fisher et al. result and assuming the use of the Nidever et al. (2002) volume *limited* sample the following is noticed (Fuhrmann 2003, pers. comm.) when comparing it to the volume *complete* Fuhrmann-sample: Of the 166 Fuhrmann stars only 90 (54%) are in Fisher et al. study despite the smaller volume but complete Fuhrmann-sample. Missing in the Fisher and Marcy (2005) study are:

1. a few subgiants,

2. fast rotating stars with  $v \sin i > 10\text{km/s}$ ,
3. a few young and chromospherically active stars,
4. stars without a precise luminosity class,
5. binaries and multiple systems,

These are all properties that make planet hunting more difficult. From these findings of Fuhrmann, I can only conclude that stars that are unfavourable for planet hunting are under-represented. The average  $[\text{Fe}/\text{H}] = -0.02$ , of the 90 Fisher et al. stars, that fall into the intersection with the Fuhrmann-sample is only 0.01 dex higher than Fuhrmann's respective value. However, for  $[\text{Fe}/\text{H}] \geq 0.2$  only 5 stars are missing but for  $[\text{Fe}/\text{H}] \leq -0.2$ , 13 stars are missing. Hence there is a slight trend that metal-poor stars are preferentially missing. Considering the still small numbers that are available for comparison, and the average metallicity of  $[\text{Fe}/\text{H}] = 0.00$ , for thin disk stars, there could be a metallicity effect of +0.10 to +0.15 (contrary to the +0.25 that are favored by Santos et al. (2003)). With the Sun at the thin disk average of  $[\text{Fe}/\text{H}]$  for our distance to the galactic Center (Fuhrmann (2004)) the role of metallicity may well be a slight increase overall due to a significant increase for the Pegasi-planet's host stars. That would put the solar system into normality as far as metallicity and planet hosting are concerned.

It is such a kind of bias, that I think might still be present in the planetary discoveries. That should be considered before abandoning what we know about the solar system.

## 27 Was the Solar System Misleading?

All things considered, was the solar system misleading? The Sun lead to much of modern physics and astrophysics. It is the calibrator of stellar evolution and the age of the universe. The Solar System offers the best traces for planet formation studies. But there are stars other than the Sun and there are planetary systems other than ours. After the dust of has settled, I think the solar system will still provide the basis to understand planetary diversity. It is not good to blame the path when getting lost!

The dominance of *strange* planets and the increased metallicity of planet hosts may be a result of hunting biases in the exoplanet sample. The close-in giant planets were not explicitly excluded by most investigators of planet formation. To a large part, the question of such planets was not considered in sufficient detail. The high eccentricities in the presently known exoplanets may be due to two effects:

1. they could represent the high-mass end of planet formation. Dynamically larger masses result in larger planet-planet interactions that typically increase the eccentricities;

2. eccentric planets might be easier to detect, because an eccentric planet excludes a wider range of neighboring planets according to Hill-exclusion stability criteria. As a result, for a sufficiently eccentric planet, there is no neighboring, competing signal, that causes confusion, thus not adding ‘planetary noise’ due to the unidentified additional radial-velocity-signals of neighboring (and smaller) planets.

We need careful studies of the observation biases, most importantly we need the study of predefined complete samples and we need an analysis of what planets may be extracted first if every planet host has a planetary system that is as dynamically filled as ours.

## 28 Observational Tests of Formation Theory

There are presently two scenarios of close-in giant planet formation theory:

**The migration scenario**, in which planets first form in a special giant planet formation region and subsequently move into orbits that are much closer to the star,

**The in-situ scenario**, where planets form near their present orbits.

All migration scenarios use violent migration as defined above with more than a factor of  $e^2$  or roughly an order of magnitude change in orbital radius from the birth-region to the final orbit. Violent migration processes as those due to disk-planet interaction, are thought to impose a severe limit on planetary life-time in the presence of the proto-planetary nebula. Yet there is apparently no evidence for such a process in the solar system. Three observational tests have been proposed to check whether violent migration is operating in general planetary systems and to distinguish between the two planet formation scenarios (Wuchterl (2001b, 2004b)):

1. the existence of Hot Neptunes,
2. birthplace exclusion in multiple stellar systems, and
3. Hill-sphere compression in volume exclusion stability criteria for planetary systems.

(1) uses the fact that migrating planets with about 10 earth masses usually accrete gas. (2) uses the fact that there is no “appropriate” birth place, e.g. beyond the ice-line, from where the planet might start out its migration, and (3) is based on the fact the Hill-spheres, and stability criteria derived from them, face a compression effect during inward migration.

Before we discuss the tests let us look at the basic assumptions used in the two scenarios. The in-situ picture assumes (1) that planets form via core-growth by planetesimal accretion, (2) there is only minor orbital evolution, and (3) that there is sufficient building material in a feeding zone near the final orbit.

Migration studies assume that (1) planets form in a special formation region, (2) there is sufficient building material in the formation region, (3) significant orbital evolution is necessary to arrive at the final planet location, and (4) migration operates and stops in time.

In situ theory predicts that giant planets can form whenever sufficient mass is available and hence there is in general no characteristic distance for their occurrence. Nebulae that provide sufficient mass in planets feeding zones are needed. That requires a diversity of protoplanetary nebulae because in the minimum mass nebula there is insufficient mass close to the star. The dynamics of envelope accretion controls what happens at the critical mass. E.g. only sufficiently convective envelopes allow accretion to Jupiter masses, and hence depending on nebula properties the mass spectrum as a function of distance is modulated.

Migration theory assumes formation beyond the snow-line, typically at 2–3 AU for a solar like star (e.g. Sasselov and Lecar (2000), see Hueso and Guillot (2003), for a discussion of the location). Planets essentially form quasi-statically whenever a critical core with a mass above a few  $M_{\text{Earth}}$  is grown or via a disk-instability, if that is possible. The latter is favoured by low temperatures and low tides, generally a large orbital distances. Planets then migrate into the present orbits and are subsequently stopped by a parking process, e.g. at the inner edge of the disk. Many of the physical processes in the migration-framework are parameterised, even for a given nebula structure and hence there are few quantitative, parameter independent predictions. But in any case the formation region should be beyond the snow-line, resulting in a minimum orbital radius,  $a_{\text{form}} > a_{\text{snow}} = 2 - 3 \text{ AU}$ . Furthermore a critical core mass of a few  $M_{\text{Earth}}$  or a gravitational instability with fragment mass of  $> 300M_{\text{Earth}}$  is necessary. The migration rates (of all types) increase towards the star, mainly because of the decrease in orbital periods and increase of nebula surface densities, cf. Equ. 38. And finally to perform the angular momentum transfer efficiently, planets need a disk of comparable mass to migrate.

## 28.1 Migration-Test I: Hot Neptunes

A Hot Neptune is a planet with supercritical mass that orbits inside the snow-line; for violent migration  $a_{\text{HN}} < a_{\text{snow}}/e^2$ . Let us assume a Hot Neptune formed in the migration scenario. Then it had to accrete first beyond the ice-line and then start migration. Migration only operates if the disk is massive, i.e. has at least comparable to the planets mass. A Hot Neptune is by definition supercritical, hence it will accrete gas if any is present Hence a migrating Neptune will continue to accrete. This does not change for type II migration because accretion can continue through gaps that have been opened by the planet in the nebula. Because the disk is still at least of comparable mass to make the planet migrate, it will accrete at least more than its own mass during the inward migration process. Hence when arriving at the final orbit, near the star it will have doubled its mass. Then it has at least two Neptune masses, or at least twice the critical mass value and is not a Hot Neptune anymore. Therefore a

Hot Neptune cannot be due to formation far out and subsequent migration.

Because we considered violent migration in the test, the planet has changed its orbital radius by much more than one feeding zone diameter and hence would have had access to many times the mass, that it accreted in its formation zone. Therefore it likely is much larger than the factor two above critical that we used in the above argument. Hence finding a Hot Neptune refutes the migration scenario because Hot Neptunes must have formed in-situ.

The recent detections of Neptune mass<sup>21</sup> planets in the planetary systems of  $\mu$  Arae (Santos et al. (2004),  $0.044 M_{\text{Jupiter}}$ , 9.55 d, 0.0955 AU), and  $\rho^1$  Cancri (McArthur et al. (2004),  $0.045 M_{\text{Jupiter}}$ , 2.81 d, 0.038 AU) easily qualify for Hot Neptunes by their orbital radii (being less than 1/10 of the ice-line) and minimum masses ( $M \sin i \approx 14 M_{\text{Earth}}$ ). Given the  $\sin i$  uncertainty there is still the unlikely possibility that their true masses could double or triple and hence weaken the test. But keeping in mind that only few systems so far have been so thoroughly observed — if any except for those two — I consider it unlikely that these  $M \sin i$  values will turn out to be outliers due to orbital projection effects and the detected planetary populations can be reconciled with migration theory.

## 28.2 Migration-Test II: The Binary Snow-Plough

This test uses the fact that the migration scenario adopts a special formation region for giant planets. Let us consider a binary system, of two equal mass components for simplicity. Then the formation region is defined to lie outside the snow-line, i.e. at orbital distances  $a > a_{\text{snow}}$  from the respective component. The gravitational sphere of influence of each stellar component reaches to the midpoint in our system, and to the respective  $L_1$  point between the components in general. If we consider subsequently closer binary systems there will be a separation where the gravitational sphere of influence of each of the components does not reach beyond the snow-line, because that orbital distance is already within the gravitational reach of the other component, beyond the  $L_1$  point in general. In such a system there is no planet formation region beyond the snow line because there exist no more orbits around the respective component. Distances beyond the snow line belong to the other component. If a planet would be found in orbit around a binary component where the formation region is taken away by the gravitational reach of the other component, the respective planet cannot have been formed in that special formation region and subsequently migrated to its detected orbit, because the formation region does not exist. Hence the migration scenario could then be ruled out.

In practice, the restrictions for the formation regions are more severe because orbits near the  $L_1$  are unstable, Dvorak (1986), Holman and Wiegert (1999). While planets with such S-type orbits are known in binary systems (e.g.  $\gamma$ -Cep as discussed earlier) the respective binary orbits are not tight enough to perform the test. But this shows that planet searches in binary systems could provide

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<sup>21</sup> $M_{\text{Neptune}} \approx 14.6 M_{\text{Earth}} \approx 0.046 M_{\text{Jupiter}}$ ,  $M_{\text{Uranus}} \approx 17.2 M_{\text{Earth}} \approx 0.054 M_{\text{Jupiter}}$

key-information about the formation processes. Thus the binary snow-plough might well remove the formation regions beyond the ice-line that are required in migration scenarios.

### 28.3 Migration-Test III: Hill-Sphere Expansion

A key result for the solar system is that it is dynamically full, Lecar et al. (2001). That means that there is no space to introduce additional planets because there are no additional stable orbits. This can be roughly understood by using a criterion that requires a minimum orbital spacing between planets, such that their mutual interactions are limited over a certain time span. These criteria can be expressed in terms of the gravitational sphere of influence of a planet, approximated by their Hill-spheres that generate a torus with radius  $R_{\text{Hill}}$  (Equ. 25) around the planets' orbit. This approximate criterion requires that certain multiples of the Hill-spheres of two planets must never touch. I.e., for any two planets,  $i$ , and  $i+1$ , with orbital radii,  $a_i, a_{i+1}$  and masses  $M_i, M_{i+1}$ ,

$$|a_i - a_{i+1}| > n [r_{\text{Hill}}(a_i, M_i) + r_{\text{Hill}}(a_{i+1}, M_{i+1})], \quad (40)$$

must hold to ensure stability of a planetary system<sup>22</sup>. The number  $n$  depends on the time-scale considered, and lies between 4 and 15. The case of two planets is well investigated now and  $n = 2 \dots 4$  seems to guarantee permanent stability (see Lecar et al. (2001)). For the solar system  $n = 13$  seems to guarantee stability over a few *Ga*.  $n=15$  might be required in some resonant systems.

If extrasolar planetary systems are dynamically full, as the solar system, their planetary spacing must obey the criterion with some  $n$ . That the systems are full seems to be indicated by the rather dense systems of massive planets in the systems discovered so far, that are often *on the edge*, Barnes and Quinn (2004), meaning that the observed orbital parameters are close to many unstable ones.

Hence the Hill exclusion criteria will be closely matched, in other words the systems will be dense. On the other hand migration rates increase with decreasing orbital distance, typically with orbital velocity, i.e.  $\propto r\Omega_{\text{Kepler}} = a^{-1/2}$ , cf. Eqns. 38,39.

If the systems form in a special formation region, and planet formation is prolific, producing full systems, the systems should obey the Hill-exclusion criterion in their formation region. But then violent migration comes into play and the innermost planets migrate away from the ones formed further out with increasing migration rates. Let us assume a system is formed in the formation region and the planetary spacing,  $\Delta a_0$  at an orbital radius,  $a_0$  is  $\Delta a_0 = nR_{\text{Hill}}(a_0, M) = na_0(M/(3M_{\odot}))^{1/3}$ . Then the system migrates inward by a factor  $1/e^2$  in radius, because that is violent migration by definition. Let us look at the system at the moment when a planet from  $a_0$  has arrived at its new orbit with  $a = a_0/e^2$ . For small spacings  $\Delta a_0/a_0 < 1$  we find for the spacing,

<sup>22</sup>This can be generalised by requiring it to hold for  $a_i(1 - e_i)$  and  $a_{i+1}(1 + e_{i+1})$

$\Delta a$  after migration to the new orbital radius,  $a$ :

$$\frac{\Delta a}{\Delta a_0} = \left(\frac{a_0}{a}\right)^{\frac{1}{2}}, \quad (41)$$

which equals  $e$  for a violent migration shrinking of an orbit by  $1/e^2$ . If we keep the masses constant during migration we can express the ratio of orbital separation between two neighbouring planets in multiples  $n, n_0$  of the respective Hill-Radii,  $R_{\text{Hill}} \propto a$ , as

$$\frac{n}{n_0} = \left(\frac{a_0}{a}\right)^{\frac{3}{2}}. \quad (42)$$

That means that a system satisfying a Hill-exclusion criterion with planets that are spaced by  $n_0 R_{\text{Hill}}(M, a_0)$  that migrates by a factor  $a_0/a = 10$  to a new orbital radius  $a$  will have a spacing of  $n R_{\text{Hill}}(M, a)$ , with  $n/n_0 = 10^{3/2} \approx 32$ ! That means that the planets are a factor 32 more widely dynamically spaced than the original system.

Since migrating planets must accrete, there is a counteracting effect due to the expansion of the Hill-spheres with increasing mass. But that is comparatively weak  $\propto M^1/3$  and should not change the conclusion.

With migration invoked for Pegasi-planets at 0.05 AU that supposedly migrated by a factor  $\sim 100$ , the effect is even more dramatic. Migration dynamically dilutes planetary systems dramatically and migrated systems would easily satisfy Hill-exclusion principles even when they started out dynamically full, just marginally obeying Hill-exclusion.

In summary, if all systems form dynamically full as the solar system and as the planetesimal accretion process seems to predict, then migrated systems must appear dynamically very underdense. If they form dynamically dense in their formation region, they will appear much wider spaced than a dynamically full system after they migrated. Hence migrated systems should be widely spaced as counted by Hill-radii, whereas in-situ formed systems should appear closely spaced, just satisfying the respective Hill-exclusion principle relevant for their age. Close-in dynamically dense observed planetary systems therefore refute systems with violent migration. Of course that assumes that violent migration processes of types I,II and III dominate more subtle dynamical planetary interactions that govern much of the later evolution, but that is after all the hypothesis of the migration origin of planets.

## 29 Planetendämmerung

To close, let us imagine we would start astronomy and wait for the stars appearing in the evening for the first time. Located at a Canary Island beach an hour or so after sunset, we might only be able to get to a visual limiting magnitude of  $\sim 3$ , say. Imagine, that our entire knowledge of stellar astronomy would have to be derived from, at first, the ten brightest stars, then the 100 brightest stars, that are the first to become visible. What would be the typical

mean stellar radius, that we would deduce for our nascent stellar astronomy? The answers — taken from the *Catalogue of Apparent Diameters and Absolute Radii of Stars*, are:

- for the 10 brightest stars the average radius is  $36 R_{\odot}$ , and
- for the 100 brightest stars the average radius is  $32 R_{\odot}$ .

This is not the astrophysics that we know! Our galaxy is dominated by low mass M-dwarfs, with radii smaller than the Sun. They outnumber the giants by large numbers. It is just that the stellar majority is hard to find and hard to see at the first glance. Similarly, I think, we will see more planets ‘appear’ on the sky! Maybe they will be more like the solar system planets.

### 30 Towards a Broad View

The next steps towards developing a broader picture into which the solar system and the extrasolar planets fit are:

1. the development of a general theory of planet formation for a diversity of protoplanetary nebulae,
2. instruments that are sensitive to the entire mass spectrum of the giant planets and below. The French-lead European COROT-mission with German participation via the DLR, should reach that goal for the first time if the planned launch in summer 2006 is successful.
3. In autumn 2007, the first jovian year will have passed since the discovery of 51 Peg b and many of the searches should then have the first complete orbits of Jupiter like planets in their data.

Finally we will have the required sensitivity for a full Jovian orbit and the theory significantly advanced. Theoretical work is ongoing in preparation for that events. Unlike with 51 Peg b, this time theory will be prepared.

### 31 Conclusion

Are the first 100 exoplanets misleading?

After more than 2500 years of astronomy we have crossed the ocean of space between us and the stars. New worlds become detectable and in reach of analysis. What has been discovered was unexpected in detail but has shown that planets are abundant in the galaxy as expected. Much can be understood on the basis of what has been learned about planet formation from the solar system, but many problems remain.

After crossing an ocean the first cliff leads to land, but don't try to get to close to it.

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